VOLUMENES DE REVOLUCION

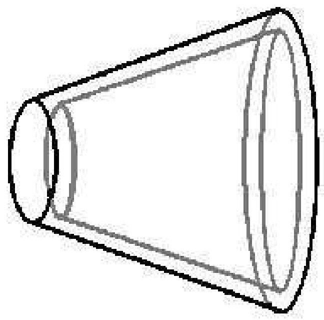
1. Calcular el volumen de revolución engendrado por la superficie limitada entre la hipérbola $x \cdot y = 3$ con la circunferencia $x^2 + y^2 = 10$, en el primer cuadrante. Dibujarlo.

2. La superficie del primer cuadrante comprendida entre $f(x) = \frac{1}{x+1}$, su recta tangente en $x = 0$ y la recta $x = 2$, gira alrededor del eje OX. Hallar el volumen del sólido generado. Dibujarlo 1 un. = 1cm.

3. Encontrar el volumen del cuerpo engendrado al girar alrededor de OX la superficie comprendida entre la hipérbola $x^2 - y^2 = a^2$ y las rectas $x = a$ y $x = 2a$

4. Hallar el volumen de la figura engendrada por $f(x) = x^2 - 4x$ al girar alrededor del e je OX

7. Calcula los cm^3 necesarios para construir el vaso de la figura.
Radios Interiores: 2 cm. y 5 cm.
Radios Exteriores: 3 cm. y 6 cm.
Altura Interior: 10 cm
Altura Exterior: 12 cm



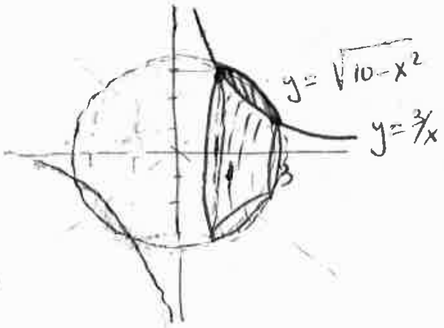
6. Calcula el volumen engendrado por la rotación alrededor del eje X de la región encerrado por la gráfica de: $y = \frac{1}{2} + \cos x$ el eje de abscisas y las rectas: $x = 0$ $x = \pi$

① $y = \frac{3}{x}$, $x^2+y^2=10$

$x^2 + \frac{9}{x^2} = 10$; $x^4+9=10x^2$

$x^4-10x^2+9=0$

$x^2 = \frac{10\pm\sqrt{100-36}}{2} = \frac{10\pm 8}{2}$ ↗ $9 \Rightarrow x=-3 \Rightarrow y=-1$; $\Rightarrow x=3 \Rightarrow y=1$ ↘ $1 \Rightarrow x=-1 \Rightarrow y=-3$; $\Rightarrow x=1 \Rightarrow y=3$



$$V = \pi\int_1^3 (\sqrt{10-x^2})^2 - \left(\frac{3}{x}\right)^2 dx =$$

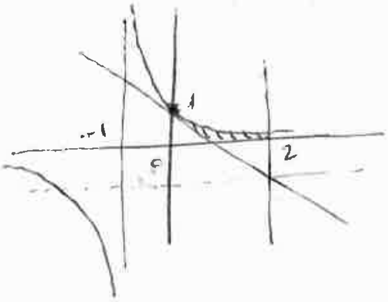
$$= \pi\int_1^3 \left(10 - x^2 - \frac{9}{x^2}\right) dx = \pi\left(10x - \frac{x^3}{3} + \frac{9}{x}\right)\Big|_1^3 =$$

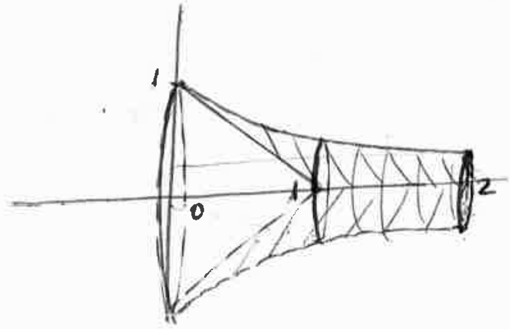
$$= \pi\left(30 - 9 + 3 - 10 + \frac{1}{3} - 9\right) = \boxed{\frac{4\pi}{3}\ uv}$$

② $y = \frac{1}{x+1}$ $\quad y' = \frac{-1}{(x+1)^2}$

$x=0 \Rightarrow y=1$ $\quad y-1 = -1(x-0) \Rightarrow \boxed{y = 1-x}$

$\Rightarrow y' = -1$



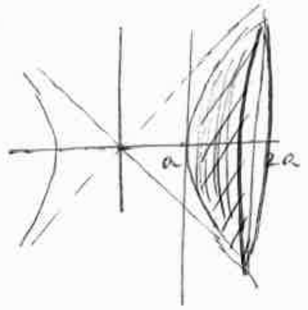


$$V = \pi\int_0^1 \left[\left(\frac{1}{x+1}\right)^2 - (1-x)^2\right] dx + \pi\int_1^2 \left(\frac{1}{x+1}\right)^2 dx =$$

$$= \pi\int_0^1 \left[\frac{1}{(x+1)^2} - (1-2x+x^2)\right] dx + \pi\int_1^2 \frac{1}{(x+1)^2} dx =$$

$$= \pi\left(\frac{-1}{x+1} - x + x^2 - \frac{x^3}{3}\right)\Big|_0^1 + \pi\,\frac{-1}{x+1}\Big|_1^2 = \pi\left(-\frac{1}{2} - 1 + 1 - \frac{1}{3} + 1\right) + \pi\left(\frac{-1}{3} + \frac{1}{2}\right) = \boxed{\frac{\pi}{3}\ uv}$$

③

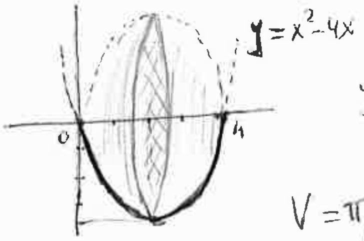


$$V = \pi\int_a^{2a}(x^2-a^2)\,dx = \pi\left(\frac{x^3}{3}-a^2x\right)\Bigg|_a^{2a} =$$

$$= \pi\left(\frac{8a^3}{3} - 2a^3 - \frac{a^3}{3} + a^3\right) = \pi\left(\frac{7a^3}{3} - a^3\right) =$$

$$= \boxed{\frac{4}{3}\pi a^3 \quad uv}$$

④

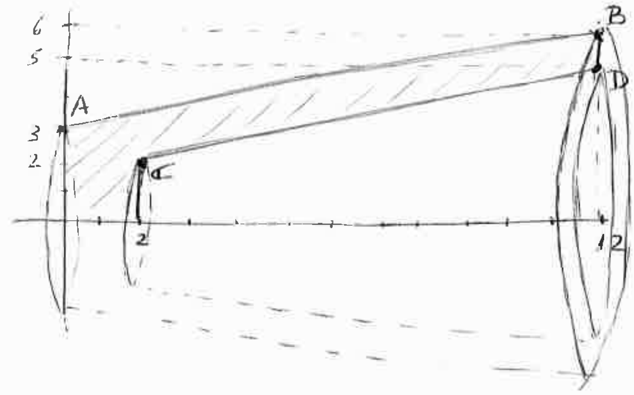


$$y=0 \Rightarrow x^2-4x=0 \quad \begin{cases} \to x=0 \\ \to x=4 \end{cases}$$

$$V = \pi\int_0^4 (x^2-4x)^2\,dx = \pi\int_0^4 (x^4-8x^3+16x^2)\,dx = \pi\left(\frac{x^5}{5} - 2x^4 + \frac{16x^3}{3}\right)\Bigg|_0^4 =$$

$$= \boxed{\frac{512\pi}{15} \quad uv}$$

⑤



$$\left.\begin{array}{l} A(0,3) \\ B(12,6) \end{array}\right| \to \vec{AB} = (12,3) \to \text{pend} = \frac{3}{12} = \frac{1}{4}$$

$$y-3 = \frac{1}{4}(x-0) \to \boxed{y = \frac{x}{4}+3}$$

$$\left.\begin{array}{l} C(2,2) \\ D(12,5) \end{array}\right| \to \vec{CD} = (10,3) \to \text{pend} = \frac{3}{10}$$

$$y-2 = \frac{3}{10}(x-2) \to y = \frac{3x}{10} - \frac{6}{10} + 2$$

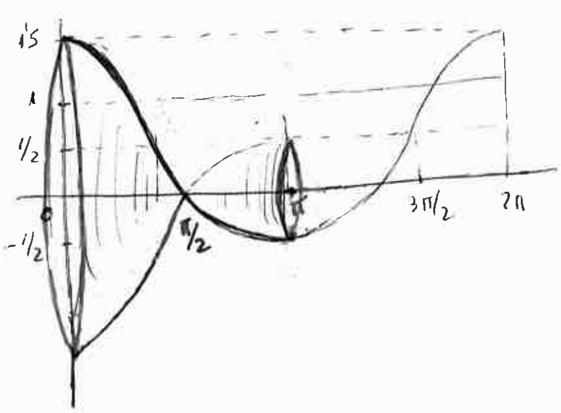
$$\boxed{y = \frac{3x+14}{10}}$$

$$V = \pi\int_0^2\left(\frac{x}{4}+3\right)^2dx + \pi\int_2^{12}\left[\left(\frac{x}{4}+3\right)^2 - \left(\frac{3x+14}{10}\right)^2\right]dx =$$

$$= \pi\int_0^{12}\left(\frac{x}{4}+3\right)^2dx - \pi\int_2^{12}\left(\frac{3x+14}{10}\right)^2dx = \pi\, 4\,\frac{\left(\frac{x}{4}+3\right)^3}{3}\Bigg|_0^{12} - \pi\,\frac{10}{3}\,\frac{\left(\frac{3x+14}{10}\right)^3}{3}\Bigg|_2^{12} =$$

$$= 288\pi - \frac{1250\pi}{9} + \frac{80\pi}{9} = \frac{1422\pi}{9} = \boxed{158\pi \quad u.v}$$

⑥



$$V = \pi\int_0^\pi\left(\frac{1}{2}+\cos x\right)^2dx = \pi\int_0^\pi\left(\frac{1}{4}+\cos x+\cos^2 x\right)dx =$$

$$= \pi\left(\frac{x}{4} + \sin x + \frac{x+\sin x\cos x}{2}\right)\Bigg|_0^\pi =$$

$$= \pi\left(\frac{\pi}{4} + 0 + \frac{\pi+0}{2}\right) - \pi\left(0+0+\frac{0+0}{2}\right) = \boxed{\frac{3\pi^2}{4} \quad uv}$$