P2

1º $y = ax + b$

(a) Calculadora: $a = 0'8050$

$b = 2'8813$

$r = 0'9777$

(b) Interpretación de a:

Por cada cliente aumenta 0'8 minutos la cola que debe guardar.

(c) $y = a \cdot 7 + b = 7 \cdot 0'8050 + 2'8813 = 8'5163$

2º $a_1 = 60$

$a_n = 60 + (n-1) \cdot (-2'5)$

$$S_n = \frac{n}{2}(a_1 + a_n) = \frac{n}{2}\left(60 + 60 + (n-1)(-2'5)\right) =$$

$$= \frac{n}{2}\left(120 - 2'5n + 2'5\right) = \frac{n}{2}\left(122'5 - 2'5n\right)$$

La función $f(x) = \frac{x}{2}(122'5 - 2'5x)$ es una parábola convexa, tendrá el máximo en el punto medio de sus raíces. Evaluamos y para $n = 24$ y $n = 25$

$S_{24} = 750 = S_{25}$ (calculadora)

3º

(a) Tabla de contingencia:

	D	$\overline{D}$	
T	8	12	20
$\overline{T}$	62	18	80
	70	30	100

(a) $p(D \cap T) = \frac{8}{100} = 0'08$

(b) $p(T \cap \overline{D}) = \frac{12}{100} = 0'12$

---

	os	as	
T	8	25% 48 = 12	20
$\overline{T}$	44	36	80
	52	48	100

c) $p(G \cap T) = \frac{12}{100} = 0'12$

(d) Serán indep.

si $p(G \cap T) = p(G) \cdot p(T)$

$p(G) = \frac{48}{100}$; $p(T) = \frac{20}{100}$

$p(G) \cdot p(T) = 0'48 \cdot 0'2 = 0'096 \neq 0'12$

No son independientes. No se cumple la fórmula.

4º

$$f(x) = 6x^2 - 12x + 1$$

$$g(x) = -x + c \qquad c \in \mathbb{R}$$

(a) $R(f) = [-5, +\infty)$ la representamos y el vértice está en $(1, -5)$.

(b) $(g \circ f)(x) \leq 0$ Determine valores de c.

$$g(f(x)) = -f(x) + c = -6x^2 + 12x - 1 + c \leq 0$$

Calculamos los valores del discriminante para los q. sólo hay una raíz, es decir $\Delta = 0$

$\Leftrightarrow b^2 - 4ac = 0 \Leftrightarrow (12)^2 - 4 \cdot (-6) \cdot (-1+c) = 0 \Leftrightarrow$

$124 + 24(-1+c) = 0$; $100 + 24c = 0 \Rightarrow c \cdot 24 = -100$

$$\boxed{c = -\frac{100}{24}}$$

5º $A(t) = A_0 \cdot e^{-kt} \quad t \geq 0 ; \quad A_0, k \in \mathbb{R}^+$

(a) $A_0 = 100$

$100 = A(0) = A_0 \cdot e^{-k \cdot 0} = A_0 \cdot 1 = A_0$

(b) $\cancel{\frac{A(t)}{2}} \quad A(5.730) = 50 \iff$

$100 \cdot e^{-k \cdot 5.730} = 50 \Rightarrow e^{-k\, 5.730} = \frac{50}{100} = 0'5$

$$\left. \begin{array}{l} \ln e^{-k \cdot 5730} = \ln\left(\frac{1}{2}\right) = \ln(2^{-1}) = -\ln(2) \\ \quad \| \\ -\ln e^{k \cdot 5730} \end{array} \right\} \Rightarrow$$

$\cancel{e}\; k \cdot 5730 = \ln(2) \Rightarrow \boxed{k = \frac{\ln(2)}{5.730}}$

(c) $A(t) = 75 \Rightarrow 100 \cdot e^{-\frac{\ln(2)}{5.730} t} = 75$

$e^{-\frac{\ln(2)}{5.730} \cdot t} = \frac{75}{100} = \frac{3}{4}$

$-\frac{\ln(2)}{5.730} \cdot t = \ln\left(\frac{3}{4}\right)$

$t = \frac{\ln(3/4) \cdot 5.730}{-\ln(2)} = 2.378'16 \simeq \underline{\underline{2.380}}$ años

6º

$$f_n(x) = \begin{cases} (n+1)\cdot x^n & 0 \le x \le 1 \\ 0 & \text{resto} \end{cases}$$

(a) $E(X] = \frac{n+1}{n+2}$

$$E[X] = \int_0^1 x \cdot (n+1)\cdot x^n \, dx = (n+1)\cdot \int_0^1 x^{n+1} dx = (n+1)\cdot \left.\frac{x^{n+2}}{n+2}\right|_0^1 =$$

$$= \frac{n+1}{(n+2)} \cdot 1^{n+2} = \frac{n+1}{n+2}$$

(b) $Var(X) = E(x^2) - [E(x)]^2 = \int_0^1 x^2 (n+1)\, x^n \, dx - \left(\frac{n+1}{n+2}\right)^2$

$$\int_0^1 x^{n+2}\cdot(n+1)\, dx = (n+1)\cdot \left.\frac{x^{n+3}}{n+3}\right|_0^1 = \frac{n+1}{n+3} \cancel{- \frac{(n+1)^2}{(n+2)^2}}$$

$$Var(X) = \frac{n+1}{n+3} - \frac{(n+1)^2}{(n+2)^2} = \frac{(n+1)\cdot(n+2)^2 - (n+3)\cdot(n+1)^2}{(n+3)\cdot(n+2)^2} =$$

$$= \frac{(n+1)\left[n^2+4n+4-(n+3)\cdot(n+1)\right]}{(n+3)\cdot(n+2)^2} = \frac{(n+1)}{(n+3)\cdot(n+2)^2}\cdot\left[n^2+4n+4-n^2-4n-3\right]$$

$$= \frac{n+1}{(n+3)\cdot(n+2)^2}$$

7º Son variaciones porque importa el orden y si participan todos son permutaciones.

(a) justo una posición después de Andrea.

$\boxed{J-A} \to$ como si fuera un corredor más.

Serían P_7 y los elementos serían:

1, 2, 3, 4, 5, 6 y $\boxed{J-A}$. $P_7 = 7! = 7\cdot 6\cdot 5\cdot 4\cdot 3\cdot 2$

$= 720 \cdot 7 = 5040$

(b) Tendríamos q. sumar.

Andrea 1ª : $P_7 = 5.040$

" 2ª : Jack puede quedar en 6 posiciones y el resto P_6

Andrea 3ª : Jack puede quedar en 5 posiciones y el resto P_6

El total sería:

$$P_7 + 6\cdot P_6 + 5\cdot P_6 + 4\cdot P_6 + 3\cdot P_6 + 2\cdot P_6 + P_6 =$$

$$P_6\cdot\left[7+6+5+4+3+2+1\right] = P_6\cdot\left[\frac{8}{\cancel{2}_{\,2}}\cdot 7\right] = 4\cdot P_7$$

$$= 4\cdot 5040 = 20160$$

8º $z = \cos\theta + i \operatorname{sen}\theta \quad , \quad z \in \mathbb{C}, \quad z \neq 1$

$$\operatorname{Re}\left(\frac{1+z}{1-z}\right) = 0$$

$$\frac{1+z}{1-z} = \frac{(1+\cos\theta) + i \operatorname{sen}\theta}{(1-\cos\theta) - i \operatorname{sen}\theta} \cdot \frac{(1-\cos\theta) + i \operatorname{sen}\theta}{(1-\cos\theta) + i \operatorname{sen}\theta} =$$

$$= \frac{(1+\cos\theta)\cdot(1-\cos\theta) - \operatorname{sen}^2\theta \quad + i\left(\qquad \right)}{(1-\cos\theta)^2 - i^2 \cdot \operatorname{sen}^2\theta}$$

$$\operatorname{Re}\left(\frac{1+z}{1-z}\right) = \frac{1-\cos^2\theta - \operatorname{sen}^2\theta}{(1-\cos\theta)^2 + \operatorname{sen}^2\theta} = \frac{0}{2\operatorname{sen}^2\theta} = 0$$

9º Por el Binomio de Newton:

$$(1+t)^{-1} = 1 - t + t^2 - t^3$$

Por el desarrollo de Mc-Laurin:

$$\cos(x) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!}$$

$$\sec x = \frac{1}{\cos x} = \frac{1}{1+(\cos x - 1)} = \left(1 + (\cos x - 1)\right)^{-1}$$

$$\cos(x) - 1 = -\frac{x^2}{2} + \frac{x^4}{24}$$

Utilizando la fórmula de (a)

$$\sec x = 1 - (\cos(x) - 1) + (\cos(x) - 1)^2 - (\cos(x) - 1)^3$$

$$= 1 - \left(-\frac{x^2}{2} + \frac{x^4}{24}\right) + \left(-\frac{x^2}{2} + \frac{x^4}{24}\right)^2 - \left(-\frac{x^2}{2} + \frac{x^4}{24}\right)^3 + \cdots$$

$$= 1 + \frac{x^2}{2} - \frac{x^4}{24} + \frac{x^4}{4} + O(x^6)$$

9º |c| $\sec(x) = 1 + \frac{x^2}{2} + \frac{5x^4}{24}$

$$\operatorname{arctg}(x) = x - \frac{x^3}{3} + \frac{x^5}{5}$$

$$\lim_{x\to 0}\left(\frac{x\cdot \operatorname{arctg}(2x)}{\sec(x) - 1}\right) = \lim_{x\to 0}\left(\frac{x\cdot\left(2x - \frac{(2x)^3}{3} + \frac{(2x)^5}{5}\right)}{1 + \frac{x^2}{2} + \frac{5x^4}{24} - 1}\right) =$$

$$= \lim_{x\to 0} \frac{x\left[2x - \frac{8}{3}x^3 + \frac{32}{5}x^5\right]}{\cancel{1} + \frac{x^2}{2} + \frac{5}{24}x^4} = \lim_{x\to 0} \frac{x^2\left[2 - \frac{8}{3}x^2 + \frac{32}{5}x^4\right]}{x^2\left[\frac{1}{2} + \frac{5}{24}\cdot x^2\right]} =$$

$$= \lim_{x\to 0} \frac{2 - \frac{8}{3}x^2 + \cdots}{1/2 + \frac{5}{24}x^2} = \frac{2}{1/2} = 4$$

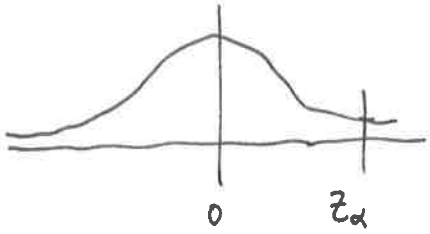
10 y 11

10: $T \equiv$ "Tiempos de vuelo"

$T \equiv N(75; \sigma)$

(a) $p(T > 82) = 0'02$

$$p\left(\frac{T-75}{\sigma} > \frac{82-75}{\sigma}\right) = p\left(z > \frac{7}{\sigma}\right) = 0'02$$



$$p(z > z_\alpha) = 0'02$$

$$z_\alpha = 2'053 \quad (\text{calculadora})$$

$$\frac{7}{\sigma} = 2'053 \implies \boxed{\sigma = \frac{7}{2'053} = 3'4084}$$

(b) $p(T > 80)$; $T \equiv N(75; 3'4084)$

$p(T>80)$ (calculadora) $\boxed{p(T>80) = 0'071169}$

(c) $$p\left(T<82 / T>80\right) = \frac{p(T<82 \cap T>80)}{p(T>80)} =$$

$$= \frac{p(80<T<82)}{p(T>80)} = \frac{0'0511}{0'071169} = 0'7191 \quad (\text{calculadora})$$

(d) Si hay 64 vuelos se espera que:

$64 \cdot p(T>80) = 64 \cdot 0'071169 = 4'55$

Entre 4 y 5 vuelos.

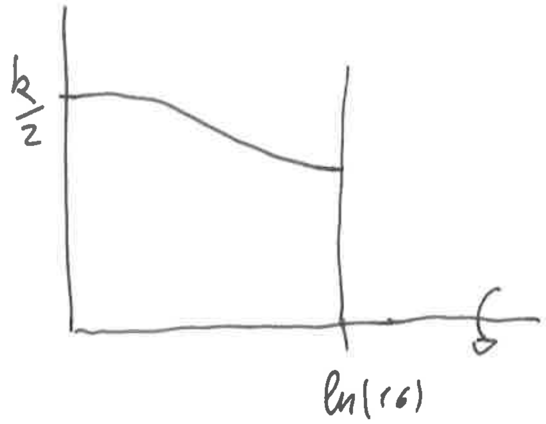
(e) $X =$ "Nº de vuelos q. duran más de 80'" $\quad X \equiv B(64; 0'071)$

$p(X>6) =$ (Calculadora con la Binomial) $= 0'1689$

(11)

(a) $f(x) = \dfrac{k e^{x/2}}{1+e^{x}}$, $x \in \mathbb{R}$ $\quad x \geq 0 \quad k \in \mathbb{R}^{+}$

(a) Volumen $= \dfrac{15k^2\pi}{34}$



Por la fórmula : $V = \displaystyle\int_0^{\ln(16)} \pi \cdot y^2\, dx = \int_0^{\ln(16)} \pi \cdot \left(\frac{k \cdot e^{x/2}}{1+e^{x}}\right)^2 dx =$

$= \displaystyle\int_0^{\ln(16)} \pi \cdot \frac{k^2\, e^{x}}{(1+e^{x})^2}\, dx = k^2 \cdot \pi \cdot \int_0^{\ln(16)} \frac{e^{x}}{(1+e^{x})^2}\, dx =$

$= k^2 \cdot \pi \cdot \displaystyle\int_1^{16} \frac{dt}{(1+t)^2} = k^2 \cdot \pi \cdot \left[-\frac{1}{(1+t)} \Big|_1^{16} \right] = \alpha$

$t = e^{x} \Rightarrow e^{0} = 1 \Rightarrow t = 1$

$e^{\ln(16)} = 16 \Rightarrow t = 16$

$dt = e^{x} dx$

$\alpha = k^2 \cdot \pi \cdot \left[-\frac{1}{17} + \frac{1}{2} \right] = k^2 \pi \cdot \left[\frac{-2+17}{34} \right] = \dfrac{15 k^2 \pi}{34}$

(b) El diseño de Pedro es la misma figura girada 90° ; por tanto el volumen coincide :

$300 = \dfrac{15k^2\pi}{34}$; $\dfrac{300 \cdot 34}{15\,\pi} = k^2$; $\quad k = \sqrt{\dfrac{20 \cdot 34}{\pi}} =$

11

(c) i) $OA = f(0) = \frac{k}{2}$

(ii) $BC = f(\ln(16)) = \frac{k \cdot e^{\frac{\ln(16)}{2}}}{1 + e^{\ln(16)}} = \frac{k \cdot \left(e^{\ln(16)}\right)^{1/2}}{1+16} =$

$= k \cdot \frac{\sqrt{16}}{17} = \frac{4k}{17}$

(d) $f(x) =$ Radio del cuerco $= \frac{k \cdot e^{x/2}}{1+e^{x}}$

$f'(x) =$ Ratio de crecimiento/Decrec.

$$f'(x) = k \cdot \left(\frac{\frac{1}{2} e^{x/2} \cdot (1+e^{x}) - e^{x/2} \cdot (1+e^{x}) \cdot e^{x}}{(1+e^{x})^{2}} \right)$$

$$= k \cdot \left(\frac{\frac{1}{2} e^{x/2} - e^{x/2} \cdot e^{x}}{(1+e^{x})} \right) = k \cdot e^{x/2} \left(\frac{\frac{1}{2} - e^{x}}{(1+e^{x})} \right)$$

$f'(x) < 0$ Es decreciente y alcanza el mínimo

en $f'(x) = 0 \Longleftrightarrow \frac{1}{2} - e^{x} = 0 \Longleftrightarrow e^{x} = \frac{1}{2}$

$$\boxed{x = \ln\left(\frac{1}{2}\right)}$$

El radio será $f\left(\ln\left(\frac{1}{2}\right)\right) = \frac{k \cdot e^{\ln(1/2)/2}}{1 + e^{\ln(1/2)}} =$

$$= \frac{k \cdot \sqrt{\frac{1}{2}}}{1 + \frac{1}{2}} = k \cdot \frac{\sqrt{1/2}}{3/2} = k \, \frac{1/\sqrt{2}}{3/2} = \frac{2k}{3\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{2\sqrt{2}\,k}{4}$$

$$= \frac{\sqrt{2}}{2} k$$

12: $f(x) = \operatorname{arc\,sen}\left(\frac{x^2-1}{x^2+1}\right)$; $x \in \mathbb{R}$

(a) $f(-x) = \operatorname{arcsen}\left(\frac{(-x)^2-1}{(-x)^2+1}\right) = \operatorname{arc\,sen}\left(\frac{x^2-1}{x^2+1}\right) = f(x)$; es par

(b) $\lim_{x\to+\infty} \operatorname{arcsen}\left(\frac{x^2-1}{x^2+1}\right) = \lim_{x\to+\infty} \{\operatorname{arcsen}(1) = \frac{\pi}{2}$

(c) (i) $f'(x) = \frac{1}{\sqrt{1-\left(\frac{x^2-1}{x^2+1}\right)^2}} \cdot \frac{2x(x^2+1)-2x(x^2-1)}{(x^2+1)^2} =$

$= \frac{1}{\sqrt{1-\frac{(x^2-1)^2}{(x^2+1)^2}}} \cdot \frac{\cancel{2x^3}+2x\cancel{-2x^3}+2x}{(x^2+1)^2} = \frac{4x}{(x^2+1)\cdot\sqrt{(x^2+1)^2\left[1-\frac{(x^2-1)^2}{(x^2+1)^2}\right]}}$

$= \frac{4x}{(x^2+1)\cdot\sqrt{(x^2+1)^2-(x^2-1)^2}} = \frac{4x}{(x^2+1)\cdot\sqrt{x^4+2x^2+1-(x^4-2x^2+1)}} =$

$= \frac{4x}{(x^2+1)\cdot\sqrt{2x^2+2x^2}} = \frac{4x}{(x^2+1)\cdot\sqrt{4x^2}} = \frac{4x}{(x^2+1)\cdot 2\sqrt{x^2}} = \frac{2x}{\sqrt{x^2}(x^2+1)}$

(ii) $\sqrt{x^2} = |x|$

Si $x<0$ $\quad f'(x) = \frac{2x}{\sqrt{x^2}\cdot(x^2+1)} < 0$ $\quad$ pues $2x<0$, $\sqrt{x^2}>0$, $(x^2+1)>0$ $\quad \frac{-}{+\cdot+} = -$

f es decreciente si $x<0$

(d) $g(x) = \arcsin\left(\dfrac{x^2-1}{x^2+1}\right)$; $x \in \mathbb{R}$ $x \geq 0$

$y = \arcsin\left(\dfrac{x^2-1}{x^2+1}\right)$; $\quad \operatorname{sen} y = \dfrac{x^2-1}{x^2+1}$;

$(\operatorname{sen} y)(x^2+1) = x^2-1$; $\quad x^2(\operatorname{sen} y - 1) + \operatorname{sen} y = -1$

$x^2 = \dfrac{-1-\operatorname{sen} y}{\operatorname{sen} y - 1}$; $\quad x^2 = \dfrac{1+\operatorname{sen} y}{1-\operatorname{sen} y}$; $\quad x = \sqrt{\dfrac{1+\operatorname{sen} y}{1-\operatorname{sen} y}}$;

$(x > 0)$ tomo $+$

$g^{-1}(x) = \sqrt{\dfrac{1+\operatorname{sen} x}{1-\operatorname{sen} x}}$

(e) $D(g^{-1}) = R(g)$

$x \geq 0$; $\quad \dfrac{1+\operatorname{sen}(x)}{1-\operatorname{sen}(x)} \geq 0 \iff \left.\begin{array}{l} 1+\operatorname{sen}(x) \geq 0 \\ 1-\operatorname{sen}(x) \geq 0 \end{array}\right\}$ Excepto $x = \frac{\pi}{2}$

$D(g^{-1}) = \mathbb{R} \setminus \left\{\frac{\pi}{2}\right\}$ que anula el denominador.

(f) $y = g^{-1}(x)$

$\lim_{x \to \pi/2} g^{-1}(x) = \lim_{x \to \pi/2} \sqrt{\dfrac{1+\operatorname{sen} x}{1-\operatorname{sen} x}} = \sqrt{\dfrac{2}{0}} = +\infty$

$\boxed{\text{A.V. en } x = \frac{\pi}{2}}$

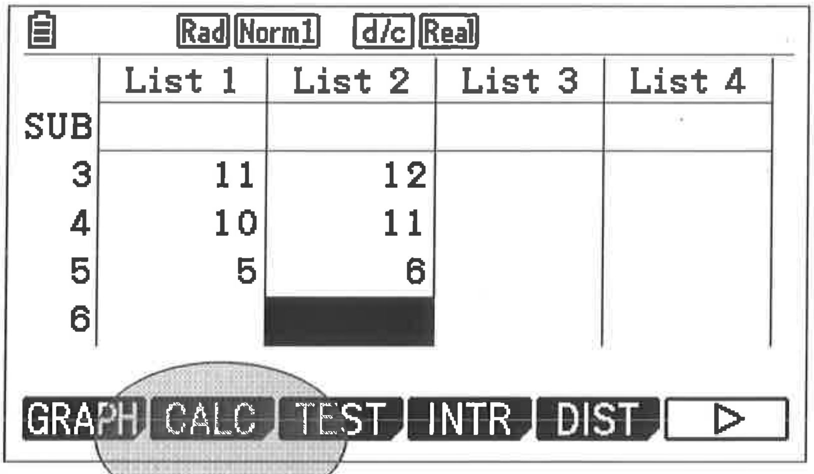
Calculadora: $\quad \operatorname{sen}(x) = -1$ corte eje X $\quad x = 3\pi/2$

$(3\pi/2, 0)$

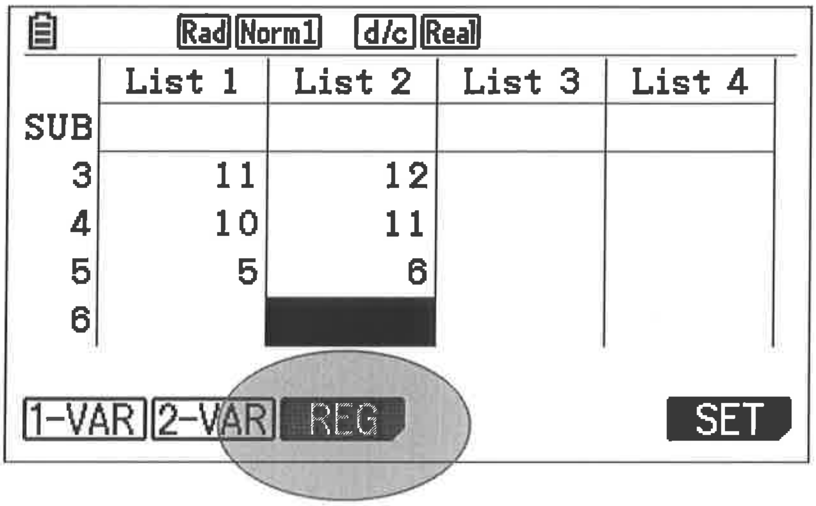
P2. Mayo 2021

Ejercicio 1

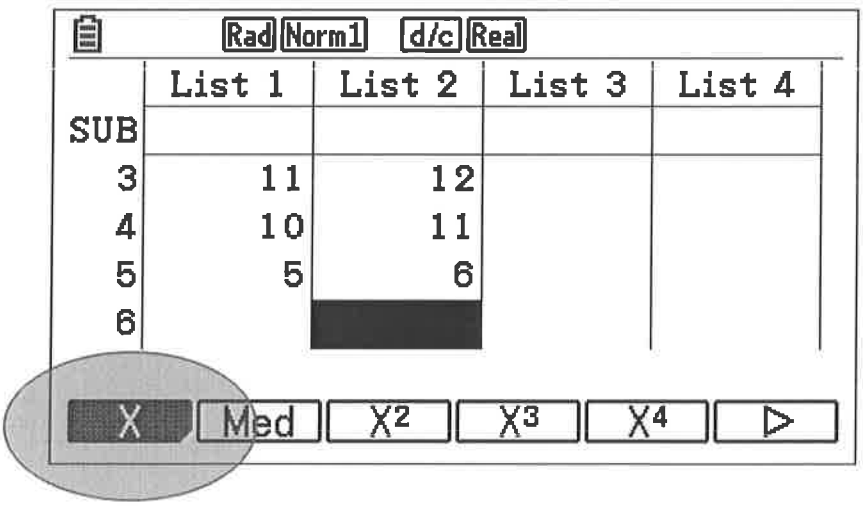
Introducimos los datos:



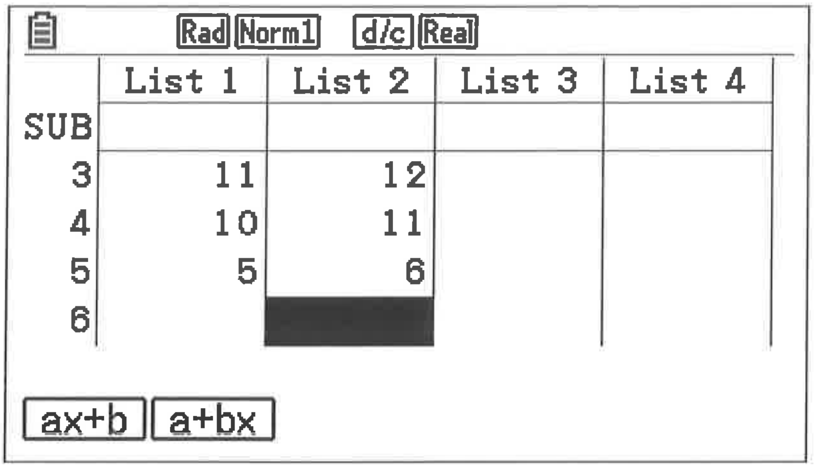
Pulsamos en F2 (calc)



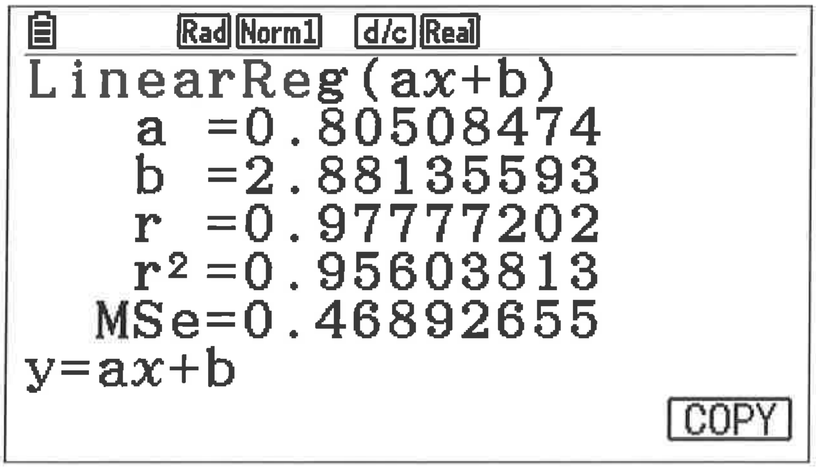
Elegimos F3 regresión



Regresión lineal F1



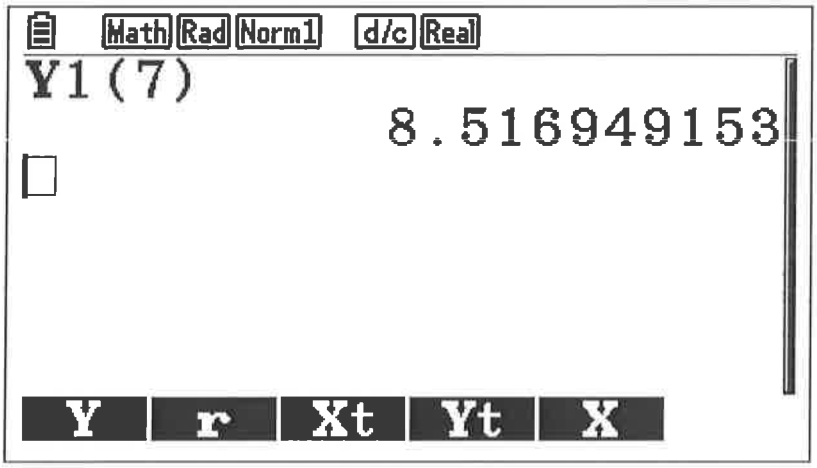
F1 recta ax + b



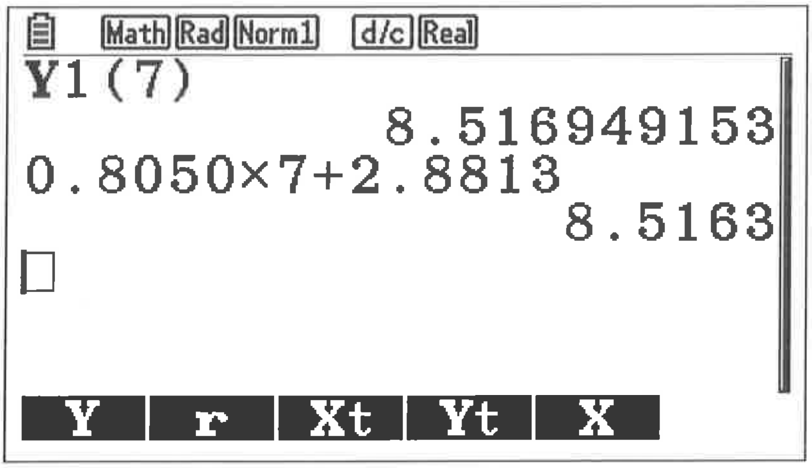
Para evaluar en el 7 vamos a menú 1

Y elegimos variables gráficas: Y1

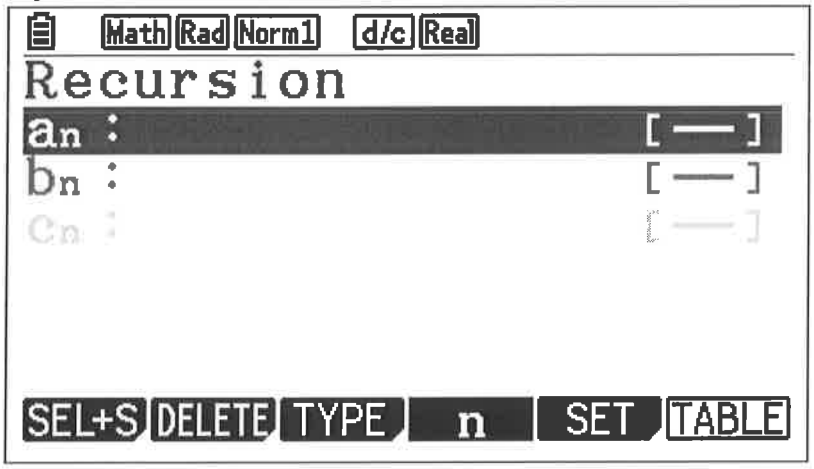
Y evaluamos en el 7



Si hacemos la operación directamente:

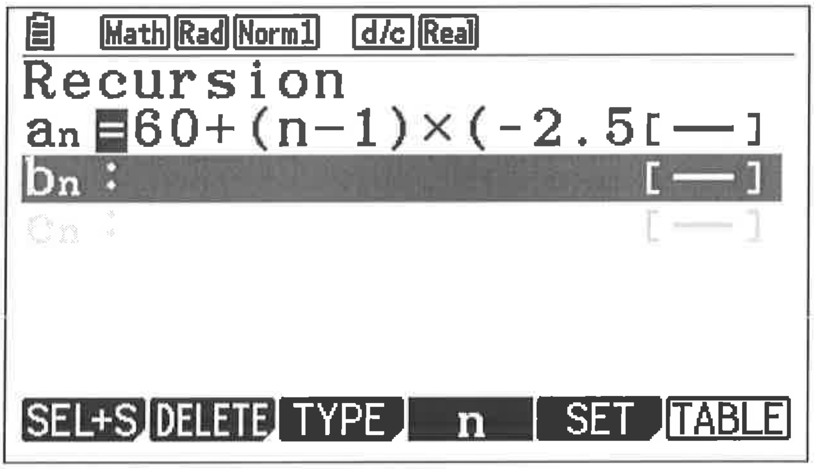


Ejercicio 2

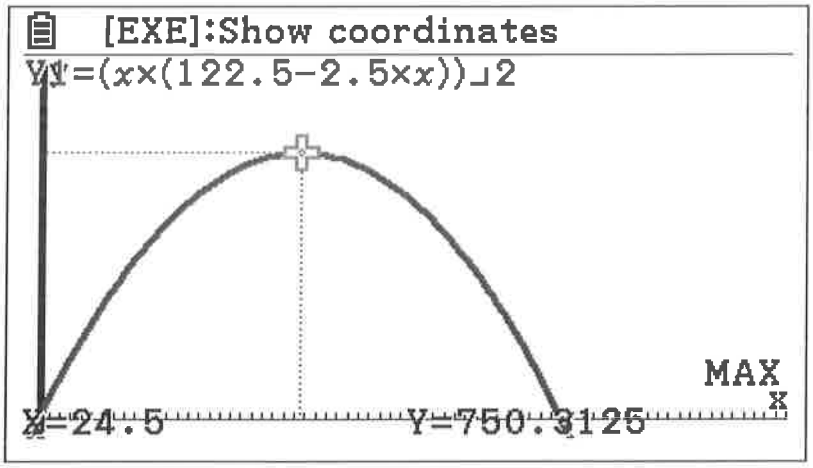


Vamos a menú 8

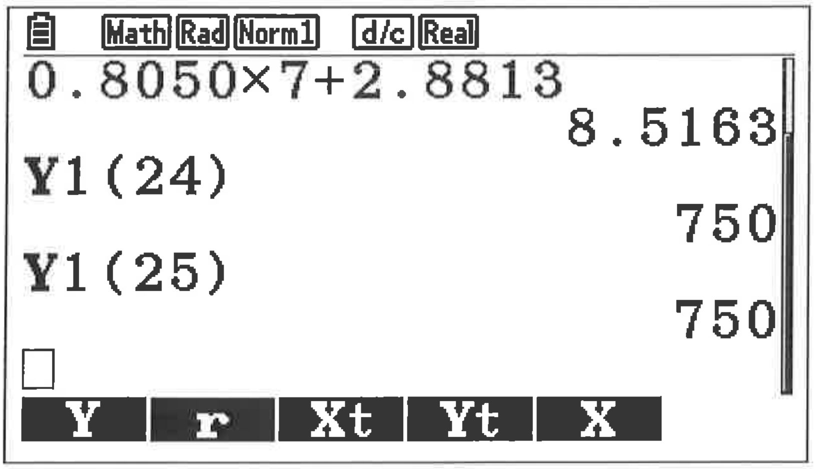
Definimos la sucesión



Dibujamos la función obtenida con la fórmula de la suma



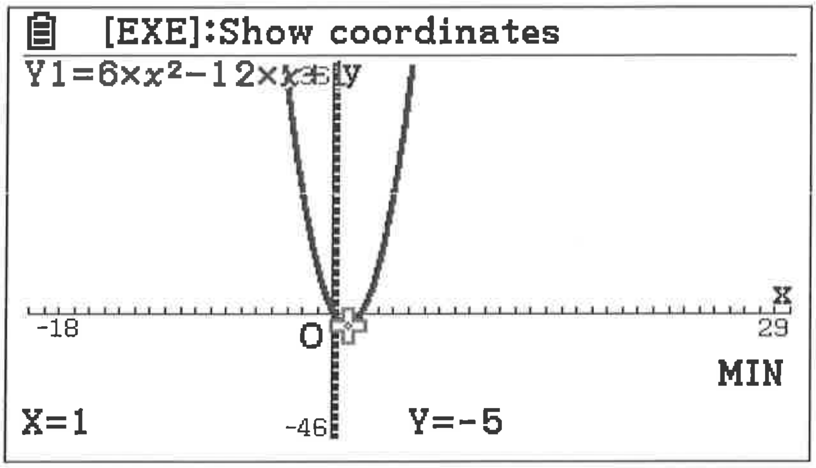
Evaluamos en el 24 y el 25



El valor máximo de la suma es 750

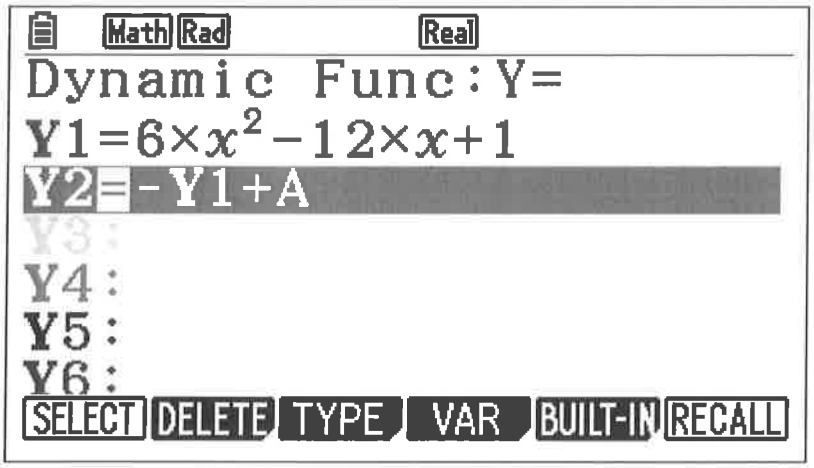
Ejercicio 4

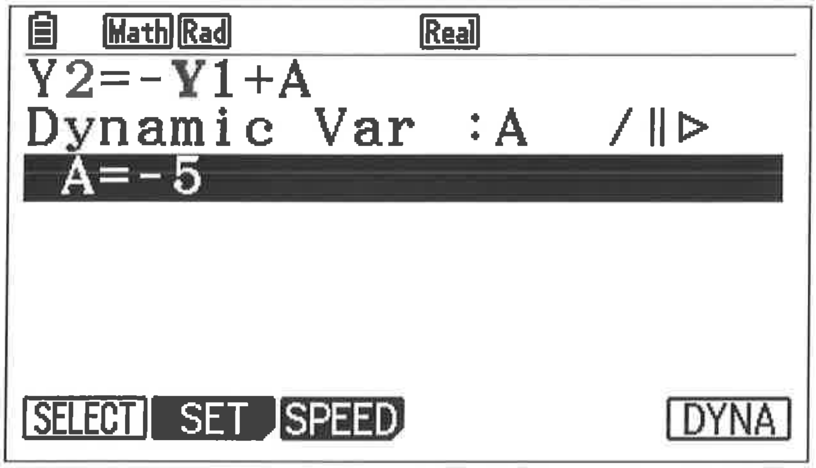
Representamos f(x)

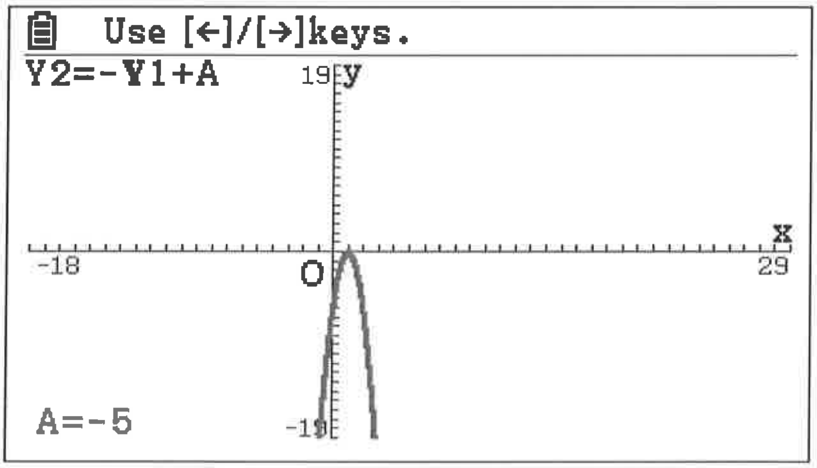


y el recorrido es [-5, +∞)

Para el apartado b podemos representar:



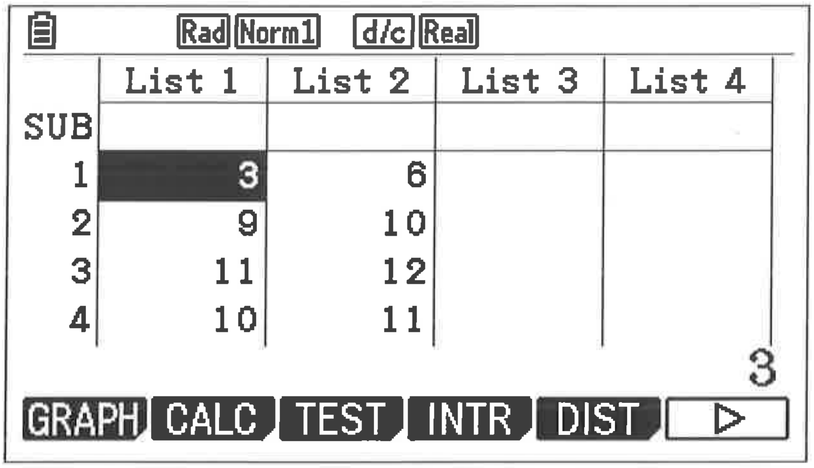




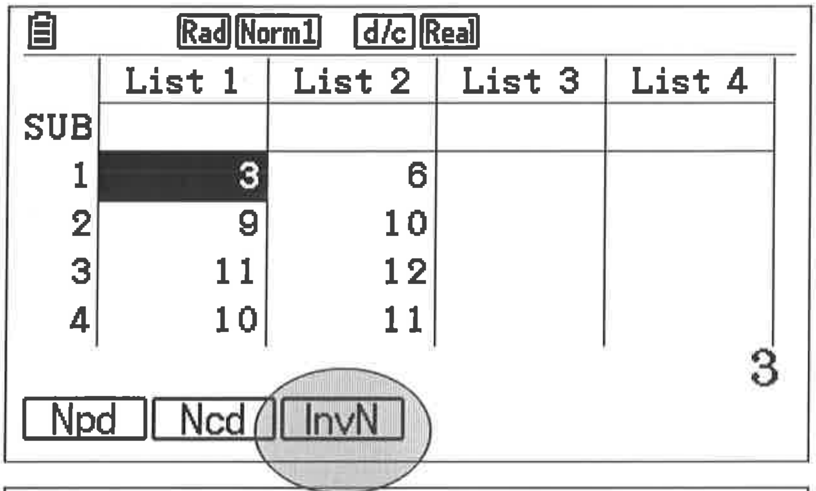
Pero es sencillo hacerlo analíticamente

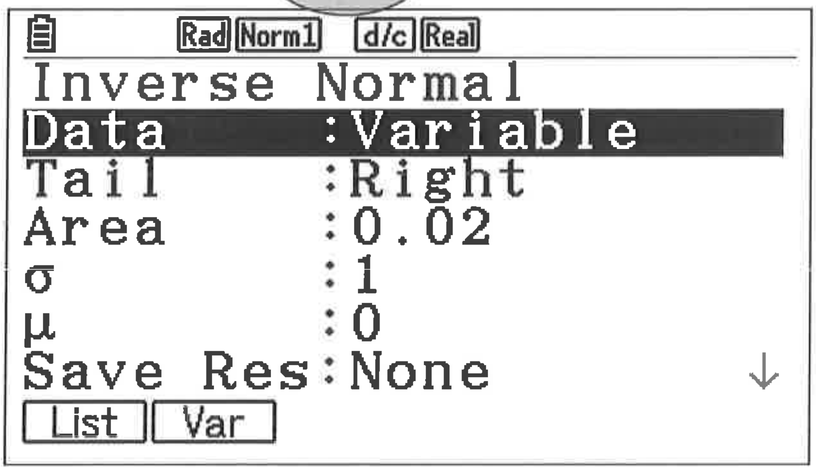
Ejercicio 10 P2 2021

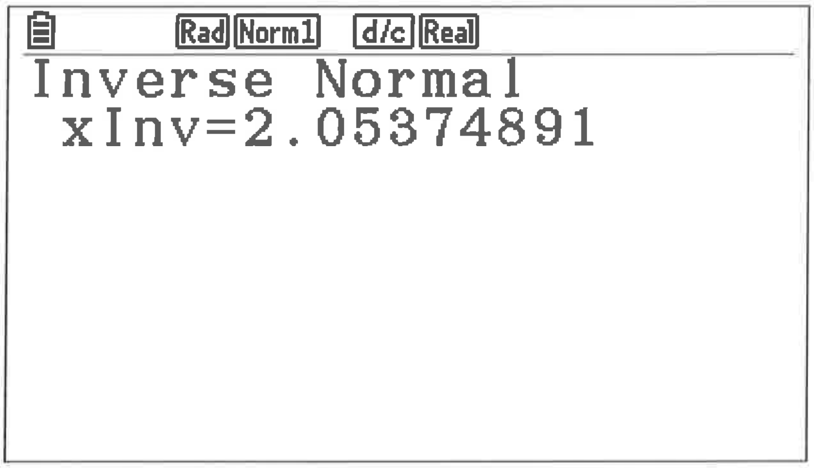
(A) Para calcular la desviación típica tipificamos y en la calculadora hallamos $p(Z<z)=0.02$



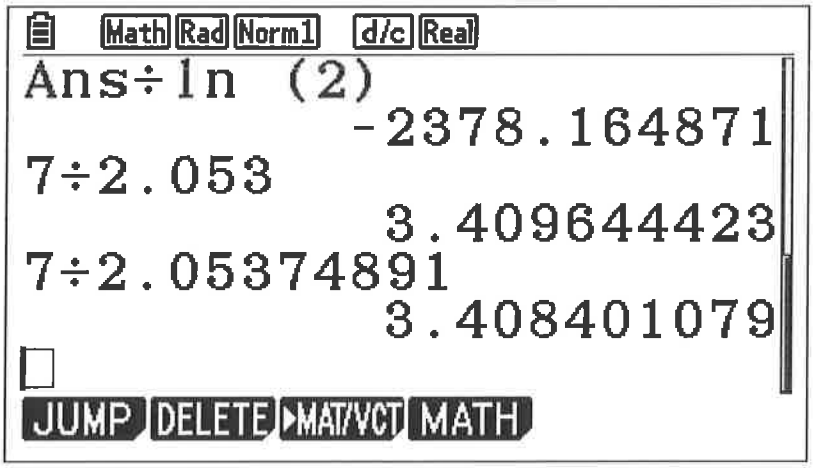
Elegimos distribución normal







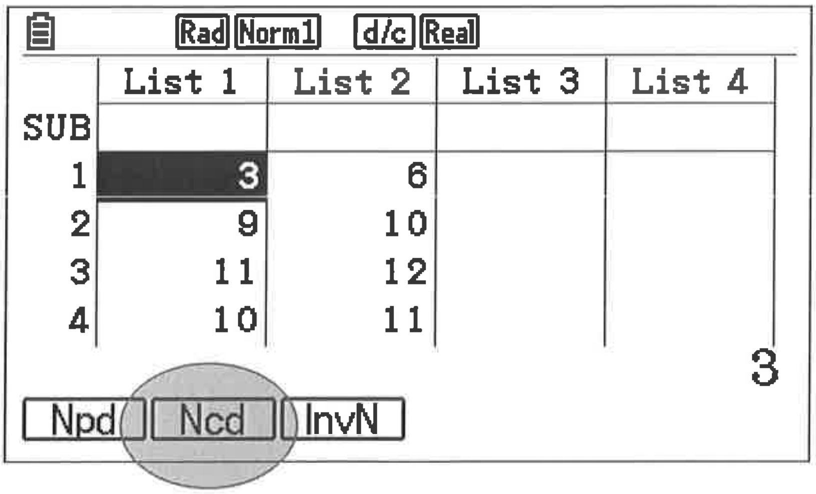
La desviación típica es 7/s= 2.053, por tanto despejando:

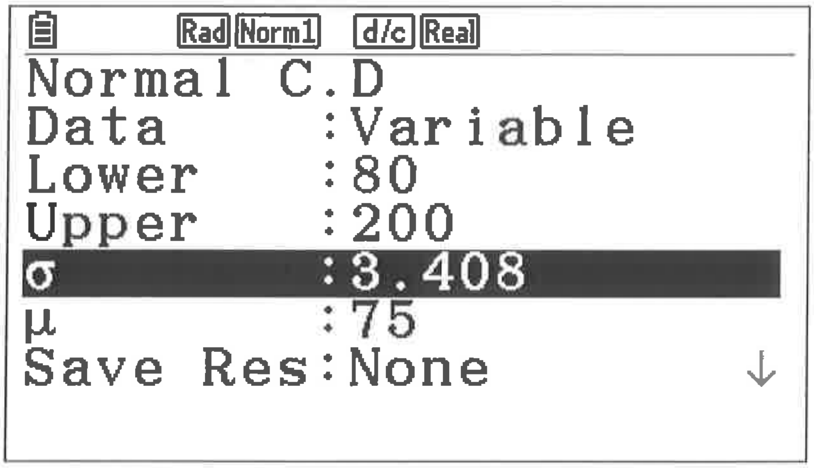


(B)

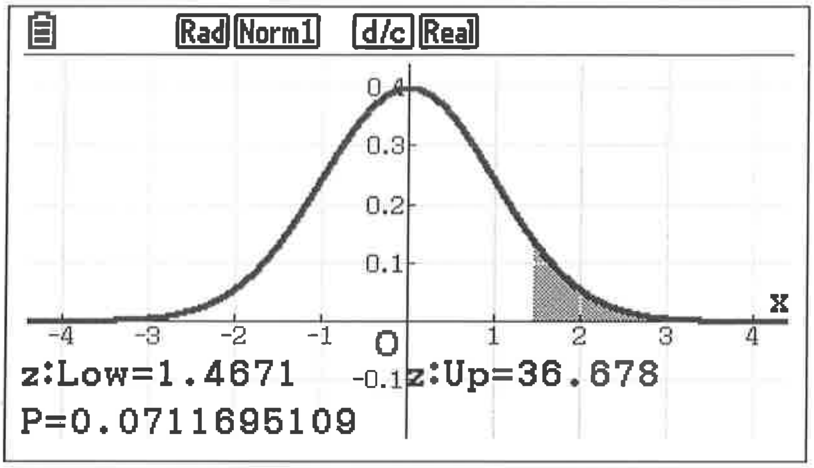
P(X>80):

Vamos a la distribución normal con el valor de la desviación típica, introducimos:





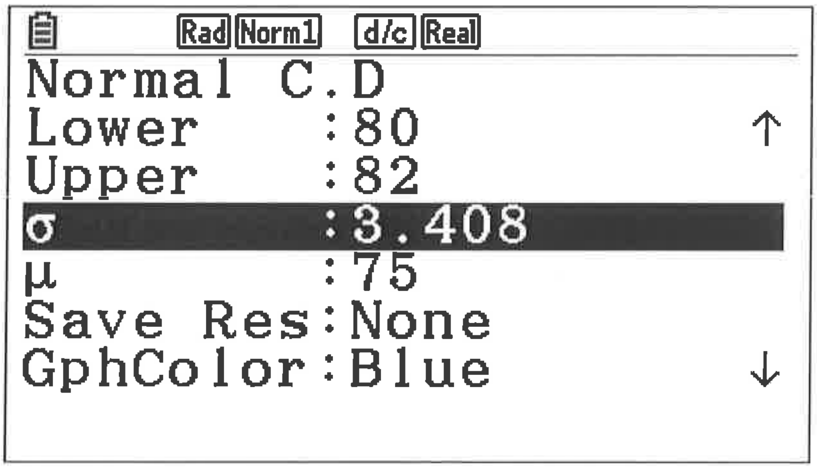
Si lo dibujamos:

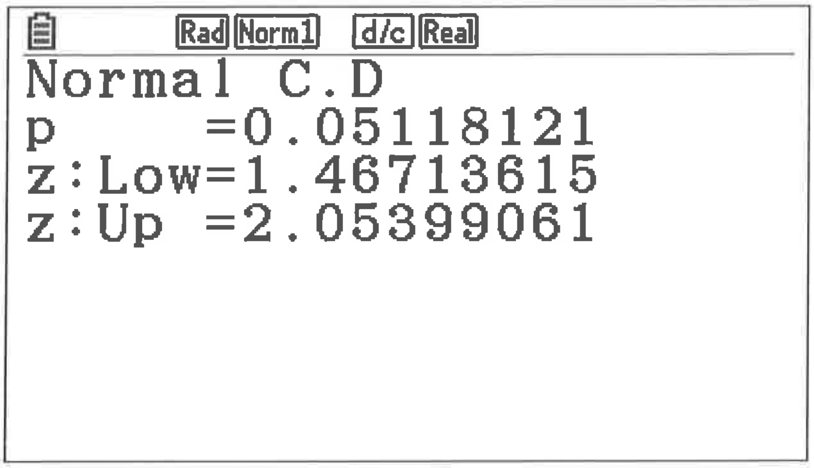


(C)

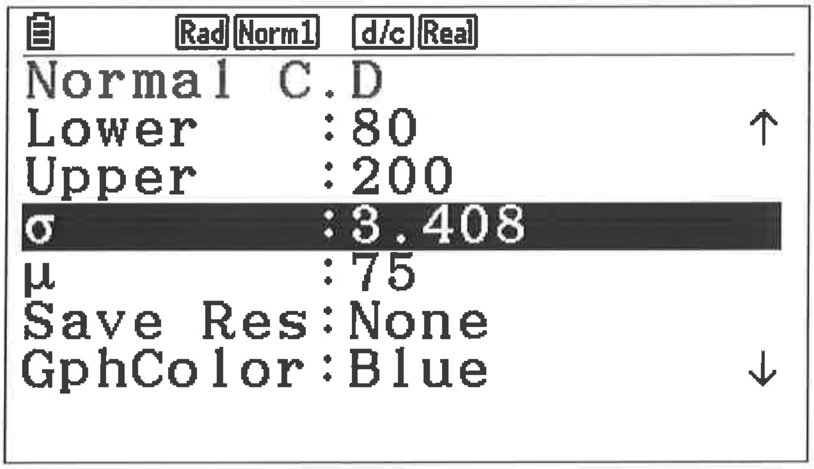
P(X<82/X>80) = P(80<X<82) / P(X<80)

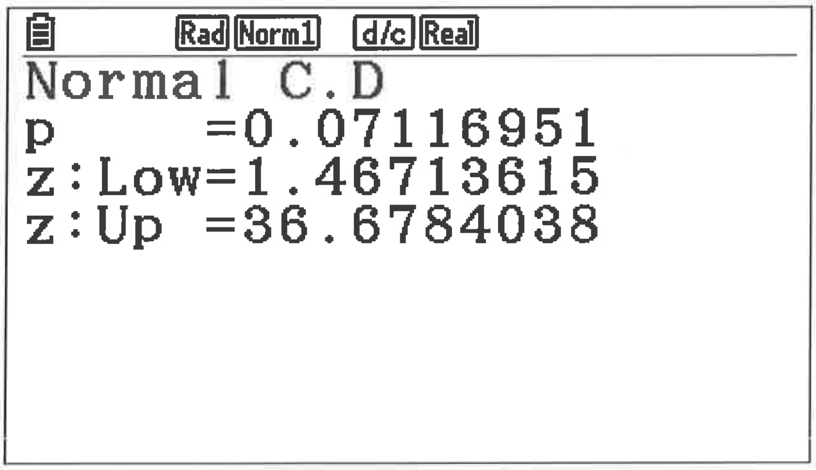
Calculamos el numerador:



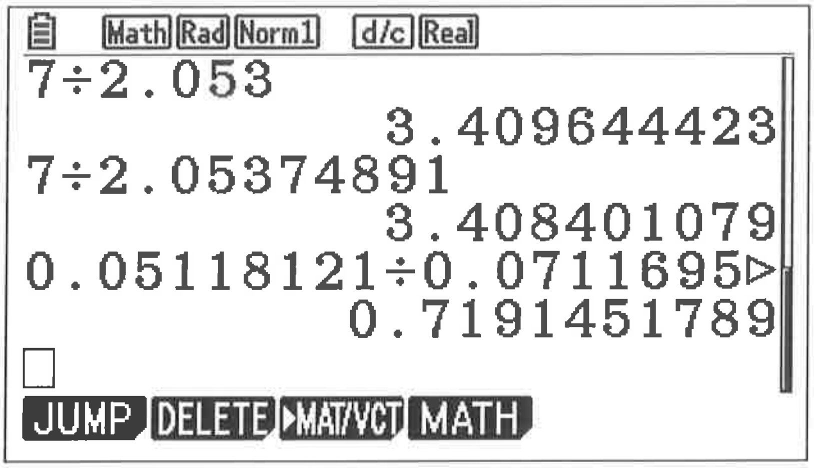


Y el denominador es el complementario del apartado (B)

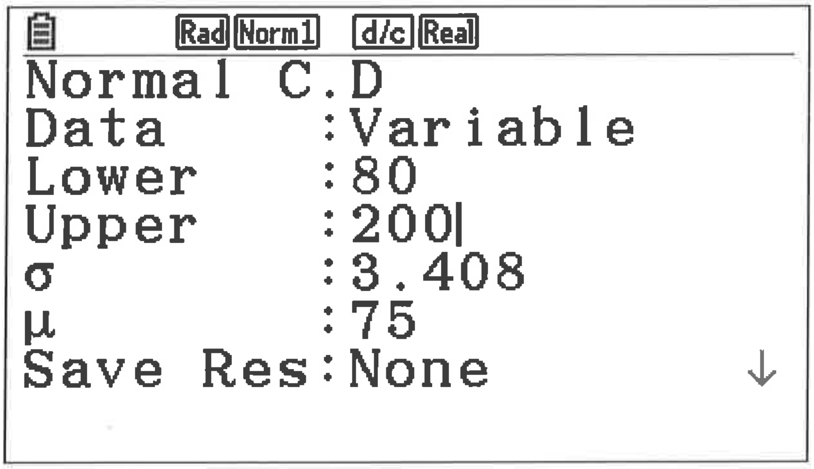


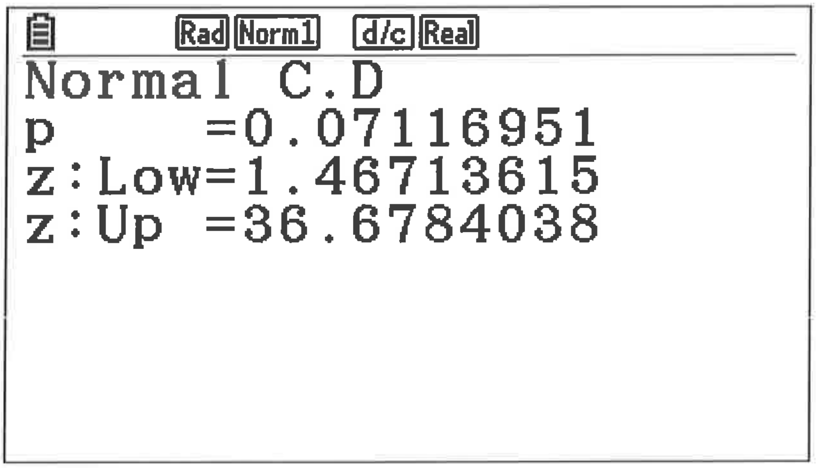


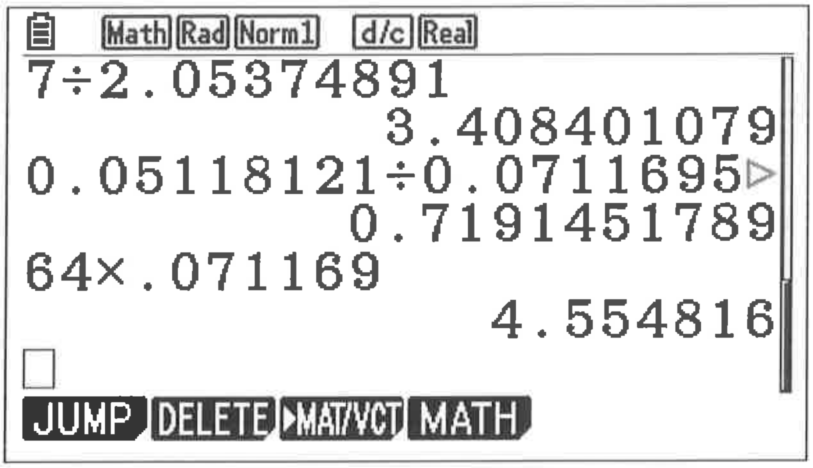
Hacemos la división:



(D) Si hay 64 vuelos se espera que haya 64* P(X>80):





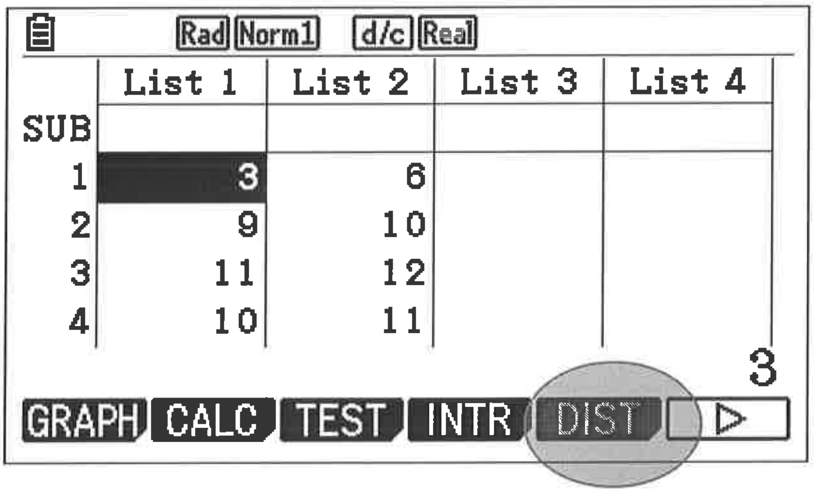


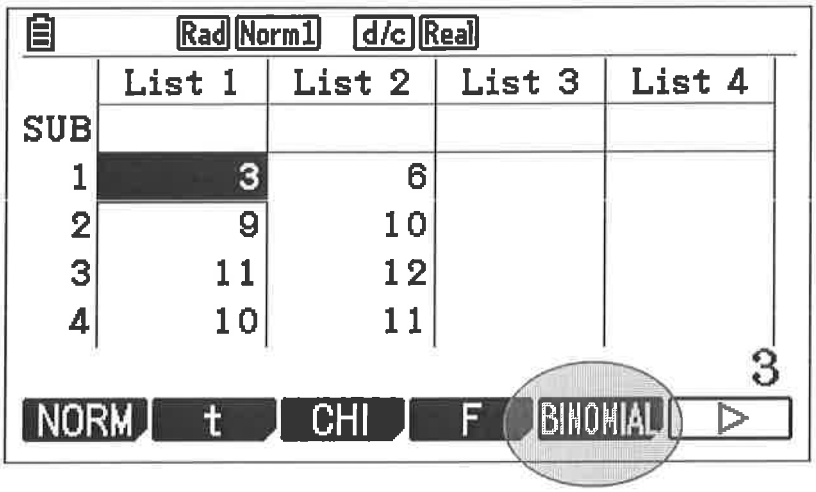
ENTRE 4 Y 5 VUELOS QUE DUREN MÁS DE 80’

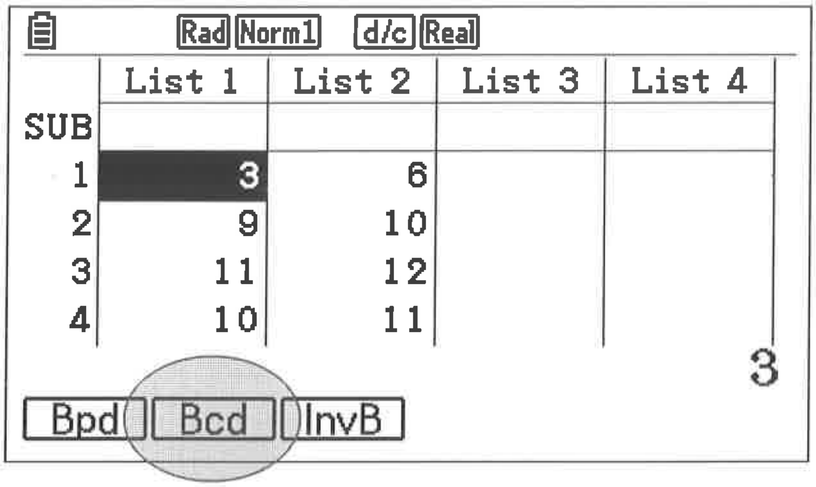
(E)

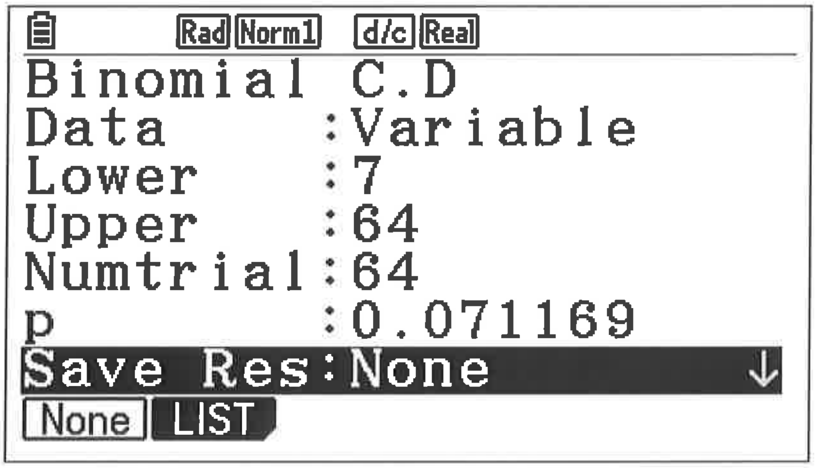
Es una binomial de n= 64 y desviación típica 0.071169

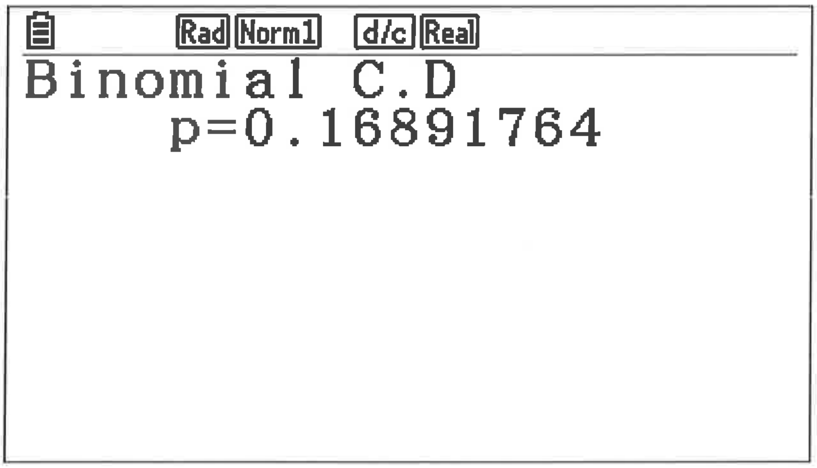
P(X>6) con la calculadora tenemos:






Rad Norm1 d/c Real
List 1 List 2 List 3 List 4
SUB
1 3 6
2 9 10
3 11 12
4 10 11
3
Bpd Bcd InvB


Rad Norm1 d/c Real
Binomial C.D
Data :Variable
Lower :7
Upper :64
Numtrial:64
p :0.071169
Save Res:None
None LIST


Rad Norm1 d/c Real
Binomial C.D
p=0.16891764