Distribuciones Continuas de Probabilidad en exámenes BI-NS

Nov 00

The lifetime of a particular component of a solar cell is Y years, where Y is a continuous random variable with probability density function

$$f(y) = \begin{cases} 0 & \text{when } y < 0 \\ 0.5e^{-y/2} & \text{when } y \geq 0. \end{cases}$$

(a) Find the probability, correct to four significant figures, that a given component fails within six months.

Each solar cell has three components which work independently and the cell will continue to run if at least two of the components continue to work.

(b) Find the probability that a solar cell fails within six months.

Nov 01

A continuous random variable X has probability density function

$$f(x) = \begin{cases} \dfrac{4}{\pi(1+x^2)}, & \text{for } 0 \leq x \leq 1, \\ 0, & \text{elsewhere.} \end{cases}$$

Find $E(X)$.

Nov 02

La función de densidad de probabilidad $f(x)$ de una variable aleatoria continua X está definida por

$$f(x) = \begin{cases} \dfrac{1}{4}x(4-x^2), & 0 \leq x \leq 2 \\ 0, & \text{para otros valores} \end{cases}$$

Calcule el valor de la **mediana** de X.

Mayo 03

Un empresario pasa X horas por día hablando por teléfono. La función de densidad de probabilidad de X está dada por

$$f(x) = \begin{cases} \dfrac{1}{12}(8x - x^3), & \text{para } 0 \leq x \leq 2 \\ 0, & \text{en los demás casos.} \end{cases}$$

(a) (i) Escriba una integral cuyo valor sea $E(X)$.

(ii) A partir de ello calcule el valor de $E(X)$.

(b) (i) Demuestre que la mediana, m, de X satisface la ecuación

$$m^4 - 16m^2 + 24 = 0.$$

(ii) A partir de ello calcule el valor de m.

(c) Calcule la moda de X.

Mayo 04

Let $f(x)$ be the probability density function for a random variable X, where

$$f(x) = \begin{cases} kx^2, & \text{for } 0 \le x \le 2 \\ 0, & \text{otherwise.} \end{cases}$$

(a) Show that $k = \frac{3}{8}$.

(b) Calculate

(i) $E(X)$;

(ii) the median of X.

Nov 04

Una variable aleatoria continua X tiene un función densidad de probabilidad dada por

$$f(x) = k(2x - x^2), \quad \text{para } 0 \le x \le 2$$
$$f(x) = 0, \quad \text{para el resto.}$$

(a) Halle el valor de k.

(b) Halle $P(0,25 \le x \le 0,5)$.

Mayo 05

(a) Use integration by parts to show that

$$\int 2x \arctan x \, dx = (x^2 + 1)\arctan x - x + C, \text{ where } C \text{ is a constant.}$$

(b) The probability density function of the random variable X is defined by

$$f(x) = \begin{cases} \frac{\pi}{2} - 2x \arctan x, & 0 \le x \le 1 \\ 0, & \text{otherwise.} \end{cases}$$

The value of a is such that $P(X < a) = \frac{3}{4}$.

(i) Show that a satisfies the equation $a(2\pi + 4) = 3 + 4(a^2 + 1)\arctan a$.

(ii) Find the value of a.

Mayo 05

La función densidad de probabilidad $f(x)$ de una variable aleatoria continua X se define en el intervalo $[0, a]$ como

$$f(x)=\begin{cases}\frac{1}{8}x & \text{para } 0\le x\le 3,\\ \frac{27}{8x^2} & \text{para } 3< x\le a.\end{cases}$$

Calcule el valor de a.

Mayo 06

The time, T minutes, required by candidates to answer a question in a mathematics examination has probability density function

$$f(t)=\begin{cases}\frac{1}{72}(12t-t^2-20), & \text{for } 4\le t\le 10\\ 0, & \text{otherwise.}\end{cases}$$

(a) Find

(i) μ, the expected value of T;

(ii) σ^2, the variance of T.

(b) A candidate is chosen at random. Find the probability that the time taken by this candidate to answer the question lies in the interval $[\mu-\sigma, \mu]$.

Nov 06

The continuous random variable X has probability density function

$$f(x)=\begin{cases}\frac{x}{1+x^2}, & \text{for } 0\le x\le k\\ 0, & \text{otherwise.}\end{cases}$$

(a) Find the exact value of k.

(b) Find the mode of X.

(c) Calculate $\mathrm{P}(1\le X\le 2)$.

Mayo 07

A continuous random variable X has probability density function f defined by

$$f(x)=\begin{cases}e^x, & \text{for } 0\le x\le \ln 2\\ 0, & \text{otherwise.}\end{cases}$$

Find the **exact** value of $\mathrm{E}(X)$.

Mayo 07 La función densidad de probabilidad f de una variable aleatoria continua X viene dada por

$$f(x)=\begin{cases}\dfrac{8}{\pi(x^2+4)}, & 0\le x\le 2\\ 0, & \text{en los demás casos.}\end{cases}$$

(a) Indiquez la moda de X.

(b) Halle el valor **exacto** de $\mathrm{E}(X)$.

Nov 07 A continuous random variable X has probability density function defined by

$$f(x)=\begin{cases}\dfrac{c}{4+x^2}, & \text{for } -\dfrac{2}{\sqrt{3}}\le x\le 2\sqrt{3}\\ 0, & \text{otherwise.}\end{cases}$$

(a) Find the **exact** value of the constant c in terms of π.

(b) Sketch the graph of $f(x)$ and hence state the mode of the distribution.

(c) Find the **exact** value of $\mathrm{E}(X)$.

Mayo 08 The random variable T has the probability density function

$$f(t)=\frac{\pi}{4}\cos\left(\frac{\pi t}{2}\right),\ -1\le t\le 1$$

Find

(a) $\mathrm{P}(T=0)$;

(b) the interquartile range.

Mayo 08 A continuous random variable X has probability density function

$$f(x)=\begin{cases}12x^2(1-x), & \text{for } 0\le x\le 1,\\ 0, & \text{otherwise.}\end{cases}$$

Find the probability that X lies between the mean and the mode.

Muestra 06/08

The continuous random variable X has probability density function

$$f(x)=\frac{1}{6}x(1+x^2) \quad \text{for } 0\le x\le 2,$$
$$f(x)=0 \quad \text{otherwise.}$$

(a) Sketch the graph of f for $0\le x\le 2$.

(b) Write down the mode of X.

(c) Find the mean of X.

(d) Find the median of X.

Muestra 08

The probability density function of the random variable X is given by

$$f(x)=\begin{cases}\dfrac{k}{\sqrt{4-x^2}}, & \text{for } 0\le x\le 1\\ 0, & \text{otherwise.}\end{cases}$$

(a) Find the value of the constant k.

(b) Show that $\mathrm{E}(X)=\dfrac{6(2-\sqrt{3})}{\pi}$.

(c) Determine whether the median of X is less than $\frac{1}{2}$ or greater than $\frac{1}{2}$

Nov 08

A continuous random variable X has a probability density function given by

$$f(x)=\begin{cases}\dfrac{(x+1)^3}{60}, & \text{for } 1\le x\le 3,\\ 0, & \text{otherwise.}\end{cases}$$

Find

(a) $\mathrm{P}(1.5\le X\le 2.5)$;

(b) $\mathrm{E}(X)$;

(c) the median of X.

Mayo 09

Una variable aleatoria tiene una función densidad de probabilidad que viene dada por

$$f(x)=\begin{cases}kx(2-x), & \text{para } 0\le x\le 2\\ 0, & \text{para los demás valores.}\end{cases}$$

(a) Compruebe que $k=\frac{3}{4}$.

(b) Halle $\mathrm{E}(X)$.

Nov 09 P2#5

The annual weather-related loss of an insurance company is modelled by a random variable X with probability density function

$$f(x)=\begin{cases}\dfrac{2.5(200)^{2.5}}{x^{3.5}}, & x\geq 200\\ 0, & \text{otherwise.}\end{cases}$$

Find the median.

Mayo 10 TZ1 P1#12

A continuous random variable X has probability density function

$$f(x)=\begin{cases}0, & x<0\\ ae^{-ax}, & x\geq 0.\end{cases}$$

It is known that $\mathrm{P}(X<1)=1-\dfrac{1}{\sqrt{2}}$.

(a) Show that $a=\dfrac{1}{2}\ln 2$.

(b) Find the median of X.

(c) Calculate the probability that $X<3$ given that $X>1$.

Mayo 10 TZ2 P1#1

A continuous random variable X has the probability density function f given by

$$f(x)=\begin{cases}c(x-x^2), & 0\leq x\leq 1\\ 0, & \text{otherwise.}\end{cases}$$

(a) Determine c.

(b) Find $\mathrm{E}(X)$.

Mayo 11 TZ2 P1#3

The random variable X has probability density function f where

$$f(x)=\begin{cases}kx(x+1)(2-x), & 0\leq x\leq 2\\ 0, & \text{otherwise.}\end{cases}$$

(a) Sketch the graph of the function. You are not required to find the coordinates of the maximum.

(b) Find the value of k.

Mayo 11 TZ1 P2#7

A continuous random variable X has a probability density function given by the function $f(x)$, where

$$f(x)=\begin{cases} k(x+2)^2, & -2\le x<0 \\ k, & 0\le x\le \frac{4}{3} \\ 0, & \text{otherwise.} \end{cases}$$

(a) Find the value of k.

(b) Hence find

(i) the mean of X;

(ii) the median of X.

Nov 11 P1#10

A continuous random variable X has the probability density function

$$f(x)=\begin{cases} k\sin x, & 0\le x\le \frac{\pi}{2} \\ 0, & \text{otherwise.} \end{cases}$$

(a) Find the value of k.

(b) Find $\mathrm{E}(X)$.

(c) Find the median of X.

Mayo 12 TZ2 P2#7

La función de densidad de probabilidad de una variable aleatoria continua X viene dada por

$$f(x)=\frac{1}{1+x^4},\ 0\le x\le a\ .$$

(a) Halle el valor de a.

(b) Halle la media de X.

Mayo 12 TZ1 P2#2

The probability density function of the continuous random variable X is given by

$$f(x)=\begin{cases} k2^{\frac{1}{x}}, & 1\le x\le 2 \\ 0, & \text{otherwise} \end{cases}$$

where k is a constant. Find the expected value of X.

Nov 12 P1#5

The continuous random variable X has probability density function given by

$$f(x)=\begin{cases} ae^{-x}, & 0\le x\le 1 \\ 0, & \text{otherwise.} \end{cases}$$

(a) State the mode of X.

(b) Determine the value of a.

(c) Find $\mathrm{E}(X)$.

Mayo 13 TZ1 P1#4

The probability density function of the random variable X is defined as

$$f(x)=\begin{cases} \sin x, & 0\le x\le \frac{\pi}{2} \\ 0, & \text{otherwise.} \end{cases}$$

Find $\mathrm{E}(X)$.

Mayo 13 TZ2 P2#7

La longitud, X metros, de los peces de una especie dada tiene la siguiente función de densidad de probabilidad:

$$f(x)=\begin{cases} ax^2, & \text{para } 0\le x\le 0{,}5 \\ 0{,}5a\,(1-x), & \text{para } 0{,}5\le x\le 1 \\ 0, & \text{resto de casos.} \end{cases}$$

(a) Compruebe que $a=9{,}6$.

(b) Dibuje aproximadamente la gráfica de la distribución.

(c) Halle $\mathrm{P}(X<0{,}6)$.