# Repaso de 1º BI-NS

## Números Reales, Polinomios, Ecuaciones, Inecuaciones e Inducción en exámenes BI-NS

**May 09 (P1)**

Prove by mathematical induction $\sum_{r=1}^{n} r(r!) = (n+1)! - 1$, $n \in \mathbb{Z}^+$

**May 09 (P2)**

(a) Show that the complex number $i$ is a root of the equation

$$x^4 - 5x^3 + 7x^2 - 5x + 6 = 0.$$

(b) Find the other roots of this equation.

**May 09 (P2)**

Let $f(x) = \dfrac{1-x}{1+x}$ and $g(x) = \sqrt{x+1}$, $x > -1$.

Find the set of values of $x$ for which $f'(x) \le f(x) \le g(x)$

**Nov 08 (P1)**

(a) Expand and simplify $(x-1)(x^4 + x^3 + x^2 + x + 1)$.

(b) Given that $b$ is a root of the equation $z^5 - 1 = 0$ which does not lie on the real axis in the Argand diagram, show that $1 + b + b^2 + b^3 + b^4 = 0$.

(c) If $u = b + b^4$ and $v = b^2 + b^3$ show that

(i) $u + v = uv = -1$;

(ii) $u - v = \sqrt{5}$, given that $u - v > 0$.

**Mayo 08 (P1)**

The polynomial $P(x) = x^3 + ax^2 + bx + 2$ is divisible by $(x+1)$ and by $(x-2)$.

Find the value of $a$ and of $b$, where $a, b \in \mathbb{R}$.

**Muestra 08 (P1)**

Solve the equation $2^{2x+2} - 10 \times 2^x + 4 = 0$, $x \in \mathbb{R}$.

**Muestra 08 (P1)**

Given that $(a + bi)^2 = 3 + 4i$ obtain a pair of simultaneous equations involving $a$ and $b$. Hence find the two square roots of $3 + 4i$.

**Muestra 08 (P1)**

Find all values of $x$ that satisfy the inequality $\dfrac{2x}{|x-1|} < 1$.

**Muestra 08 (P1)**

Use mathematical induction to prove that $5^n + 9^n + 2$ is divisible by 4, for $n \in \mathbb{Z}^+$.

## Números Complejos en exámenes BI-NS

**May 09 (P1)**

Consider the complex numbers $z = 1 + 2\mathrm{i}$ and $w = 2 + a\mathrm{i}$, where $a \in \mathbb{R}$.

Find $a$ when

(a) $|w| = 2|z|$;

(b) $\mathrm{Re}(zw) = 2\,\mathrm{Im}(zw)$.

**May 09 (P1)**

Given that $z_1 = 2$ and $z_2 = 1 + \sqrt{3}\mathrm{i}$ are roots of the cubic equation $z^3 + bz^2 + cz + d = 0$ where $b, c, d \in \mathbb{R}$,

(a) write down the third root, $z_3$, of the equation;

(b) find the values of $b$, $c$ and $d$;

(c) write $z_2$ and $z_3$ in the form $re^{\mathrm{i}\theta}$.

**May 09 (P1)**

If $z$ is a non-zero complex number, we define $L(z)$ by the equation

$$L(z) = \ln|z| + \mathrm{i}\arg(z),\ 0 \le \arg(z) < 2\pi.$$

(a) Show that when $z$ is a positive real number, $L(z) = \ln z$.

(b) Use the equation to calculate

(i) $L(-1)$;

(ii) $L(1-\mathrm{i})$;

(iii) $L(-1+\mathrm{i})$.

(c) Hence show that the property $L(z_1 z_2) = L(z_1) + L(z_2)$ does not hold for all values of $z_1$ and $z_2$.

**May 08 (P1)**

Express $\dfrac{1}{(1-\mathrm{i}\sqrt{3})^3}$ in the form $\dfrac{a}{b}$ where $a, b \in \mathbb{Z}$.

**May 08 (P2)**

$z_1 = (1 + \mathrm{i}\sqrt{3})^m$ and $z_2 = (1-\mathrm{i})^n$.

(a) Find the modulus and argument of $z_1$ and $z_2$ in terms of $m$ and $n$, respectively.

(b) **Hence**, find the smallest positive integers $m$ and $n$ such that $z_1 = z_2$.

**Muestra 08 (P1)**

Find the three cube roots of the complex number $8\mathrm{i}$. Give your answers in the form $x + \mathrm{i}y$.

## Progresiones en exámenes BI-NS

**Nov 08 (P1)**

An 81 metre rope is cut into $n$ pieces of increasing lengths that form an arithmetic sequence with a common difference of $d$ metres. Given that the lengths of the shortest and longest pieces are 1.5 metres and 7.5 metres respectively, find the values of $n$ and $d$.

**Nov 08 (P2)**

A geometric sequence has a first term of 2 and a common ratio of 1.05. Find the value of the smallest term which is greater than 500.

**Mayo 08 (P1)**

The common ratio of the terms in a geometric series is $2^x$.

(a) State the set of values of $x$ for which the sum to infinity of the series exists.

(b) If the first term of the series is 35, find the value of $x$ for which the sum to infinity is 40.

**Muestra 08 (P1)**

(a) Show that $p = 2$ is a solution to the equation $p^3 + p^2 - 5p - 2 = 0$.

(b) Find the values of $a$ and $b$ such that $p^3 + p^2 - 5p - 2 = (p-2)(p^2 + ap + b)$.

(c) Hence find the other two roots to the equation $p^3 + p^2 - 5p - 2 = 0$.

(d) An arithmetic sequence has $p$ as its common difference. Also, a geometric sequence has $p$ as its common ratio. Both sequences have 1 as their first term.

(i) Write down, in terms of $p$, the first four terms of each sequence.

(ii) If the sum of the third and fourth terms of the arithmetic sequence is equal to the sum of the third and fourth terms of the geometric sequence, find the three possible values of $p$.

(iii) For which value of $p$ found in (d)(ii) does the sum to infinity of the terms of the geometric sequence exist?

(iv) For the same value $p$, find the sum of the first 20 terms of the arithmetic sequence writing your answer in the form $a + b\sqrt{c}$, where $a, b, c \in \mathbb{Z}$.

**Muestra 08 (P2)**

A sum of $\$\,5000$ is invested at a compound interest rate of 6.3 % per annum.

(a) Write down an expression for the value of the investment after $n$ full years.

(b) What will be the value of the investment at the end of five years?

(c) The value of the investment will exceed $\$\,10\,000$ after $n$ full years.

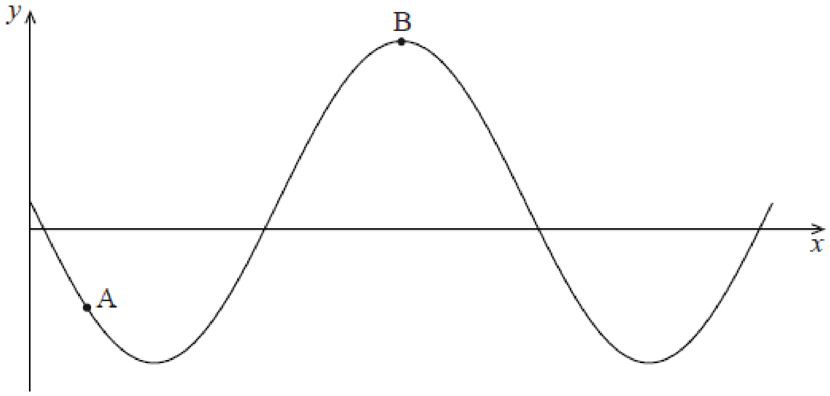
(i) Write an inequality to represent this information.

(ii) Calculate the minimum value of $n$.

## Propiedades básicas de funciones en exámenes BI-NS

**May 09 (P1)**

The diagram below shows a curve with equation $y = 1 + k\sin x$, defined for $0 \le x \le 3\pi$.




The point $A\left(\frac{\pi}{6}, -2\right)$ lies on the curve and $B(a, b)$ is the maximum point.

(a) Show that $k = -6$.

(b) Hence, find the values of $a$ and $b$.

**May 09 (P1)**

Let $g(x) = \log_5 |2\log_3 x|$. Find the product of the zeros of $g$.

**May 09 (P2)**

The graph of $y = \ln(x)$ is transformed into the graph of $y = \ln(2x+1)$. Describe two transformations that are required to do this.

**Nov 08 (P1)**

Write $\ln(x^2 - 1) - 2\ln(x+1) + \ln(x^2 + x)$ as a single logarithm, in its simplest form.

**May 08 (P2)**

The depth, $h(t)$ metres, of water at the entrance to a harbour at $t$ hours after midnight on a particular day is given by

$$h(t) = 8 + 4\sin\left(\frac{\pi t}{6}\right),\ 0 \le t \le 24.$$

(a) Find the maximum depth and the minimum depth of the water.

(b) Find the values of $t$ for which $h(t) \ge 8$.

**Muestra 08 (P1)**

The functions $f$ and $g$ are defined by $f : x \mapsto e^x$, $g : x \mapsto x + 2$.

Calculate

(a) $f^{-1}(3) \times g^{-1}(3)$;

(b) $(f \circ g)^{-1}(3)$.

## Estadística en exámenes BI-NS

**Mayo 09 (P1)**

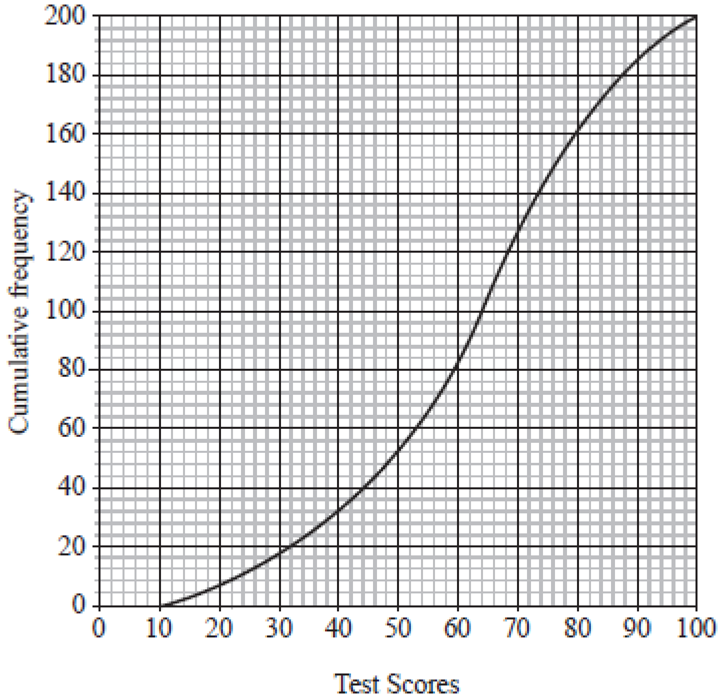
Consider now the set of numbers $x_1, \ldots, x_m, y_1, \ldots, y_n$ where $x_i = 0$ for $i = 1, \ldots, m$ and $y_i = 1$ for $i = 1, \ldots, n$.

(i) Show that the mean $M$ of this set is given by $\frac{n}{m+n}$ and the standard deviation $S$ by $\frac{\sqrt{mn}}{m+n}$.

(ii) Given that $M = S$, find the value of the median.

**Mayo 09 (P1)**

The test scores of a group of students are shown on the cumulative frequency graph below.




(a) Estimate the median test score.

(b) The top 10 % of students receive a grade A and the next best 20 % of students receive a grade B. Estimate

(i) the minimum score required to obtain a grade A;

(ii) the minimum score required to obtain a grade B.

## Probabilidad en exámenes BI-NS

**Mayo 09 (P2)**

In a class of 20 students, 12 study Biology, 15 study History and 2 students study neither Biology nor History.

(a) Illustrate this information on a Venn diagram.

(b) Find the probability that a randomly selected student from this class is studying both Biology and History.

(c) Given that a randomly selected student studies Biology, find the probability that this student also studies History.

**Mayo 09 (P2)**

An influenza virus is spreading through a city. A vaccination is available to protect against the virus. If a person has had the vaccination, the probability of catching the virus is 0.1; without the vaccination, the probability is 0.3. The probability of a randomly selected person catching the virus is 0.22.

(a) Find the percentage of the population that has been vaccinated.

(b) A randomly chosen person catches the virus. Find the probability that this person has been vaccinated.

**Mayo 08 (P1)**

Let $A$ and $B$ be events such that $\mathrm{P}(A) = 0.6$, $\mathrm{P}(A \cup B) = 0.8$ and $\mathrm{P}(A \mid B) = 0.6$.

Find $\mathrm{P}(B)$.

**Mayo 08 (P2)**

Only two international airlines fly daily into an airport. UN Air has 70 flights a day and IS Air has 65 flights a day. Passengers flying with UN Air have an 18 % probability of losing their luggage and passengers flying with IS Air have a 23 % probability of losing their luggage. You overhear someone in the airport complain about her luggage being lost.

Find the probability that she travelled with IS Air.

**Muestra 08 (P1)**

If $\mathrm{P}(A) = \frac{1}{6}$, $\mathrm{P}(B) = \frac{1}{3}$, and $\mathrm{P}(A \cup B) = \frac{5}{12}$, what is $\mathrm{P}(A' / B')$?

**Muestra 08 (P2)**

Bag A contains 2 red and 3 green balls.

(a) Two balls are chosen at random from the bag without replacement. Find the probability that 2 red balls are chosen.

Bag B contains 4 red and $n$ green balls.

(b) Two balls are chosen without replacement from this bag. If the probability that two red balls are chosen is $\frac{2}{15}$, show that $n = 6$.

A standard die with six faces is rolled. If a 1 or 6 is obtained, two balls are chosen from bag A, otherwise two balls are chosen from bag B.

(c) Calculate the probability that two red balls are chosen.

(d) Given that two red balls are chosen, find the probability that a 1 or a 6 was obtained on the die.

## Distribuciones de probabilidad en exámenes BI-NS

**Mayo 09 (P2)**

Bob measured the heights of 63 students. After analysis, he conjectured that the height, $H$, of the students could be modelled by a normal distribution with mean 166.5 cm and standard deviation 5 cm.

(a) Based on this assumption, estimate the number of these students whose height is at least 170 cm.

Later Bob noticed that the tape he had used to measure the heights was faulty as it started at the 5 cm mark and not at the zero mark.

(b) What are the correct values of the mean and variance of the distribution of the heights of these students?

**Mayo 09 (P2)**

Mr Lee is planning to go fishing this weekend. Assuming that the number of fish caught per hour follows a Poisson distribution with mean 0.6, find

(a) the probability that he catches at least one fish in the first hour;

(b) the probability that he catches exactly three fish if he fishes for four hours;

(c) the number of **complete** hours that Mr Lee needs to fish so that the probability of catching more than two fish exceeds 80 %.

**Mayo 09 (P2)**

Testing has shown that the volume of drink in a bottle of mineral water filled by **Machine A** at a bottling plant is normally distributed with a mean of 998 ml and a standard deviation of 2.5 ml.

(a) Show that the probability that a randomly selected bottle filled by Machine A contains more than 1000 ml of mineral water is 0.212.

(b) A random sample of 5 bottles is taken from Machine A. Find the probability that exactly 3 of them each contain more than 1000 ml of mineral water.

(c) Find the minimum number of bottles that would need to be sampled to ensure that the probability of getting at least one bottle filled by Machine A containing more than 1000 ml of mineral water, is greater than 0.99.

(d) It has been found that for **Machine B** the probability of a bottle containing less than 996 ml of mineral water is 0.1151. The probability of a bottle containing more than 1000 ml is 0.3446. Find the mean and standard deviation for the volume of mineral water contained in bottles filled by Machine B.

(e) The company that makes the mineral water receives, on average, $m$ phone calls every 10 minutes. The number of phone calls, $X$, follows a Poisson distribution such that $\mathrm{P}(X=2)=\mathrm{P}(X=3)+\mathrm{P}(X=4)$.

(i) Find the value of $m$.

(ii) Find the probability that the company receives more than two telephone calls in a randomly selected 10 minute period.

**Nov 08 (P1)**

John removes the labels from three cans of tomato soup and two cans of chicken soup in order to enter a competition, and puts the cans away. He then discovers that the cans are identical, so that he cannot distinguish between cans of tomato soup and chicken soup. Some weeks later he decides to have a can of chicken soup for lunch. He opens the cans at random until he opens a can of chicken soup. Let $Y$ denote the number of cans he opens.

Find

(a) the possible values of $Y$,

(b) the probability of each of these values of $Y$,

(c) the expected value of $Y$.

**Muestra 08 (P1)**

A discrete random variable $X$ has its probability distribution given by

$$P(X = x) = k(x+1), \text{ where } x \text{ is } 0, 1, 2, 3, 4.$$

(a) Show that $k = \dfrac{1}{15}$.

(b) Find $E(X)$.

**Muestra 08 (P2)**

The speeds of cars at a certain point on a straight road are normally distributed with mean $\mu$ and standard deviation $\sigma$. 15 % of the cars travelled at speeds greater than $90 \text{ km h}^{-1}$ and 12 % of them at speeds less than $40 \text{ km h}^{-1}$. Find $\mu$ and $\sigma$.

**Muestra 08 (P2)**

There are 30 students in a class, of which 18 are girls and 12 are boys. Four students are selected at random to form a committee. Calculate the probability that the committee contains

(a) two girls and two boys;

(b) students all of the same gender.

## Trigonometría en exámenes BI-NS

**May 09 (P1)**

(a) Show that $\arctan\left(\frac{1}{2}\right)+\arctan\left(\frac{1}{3}\right)=\frac{\pi}{4}$.

(b) Hence, or otherwise, find the value of $\arctan(2)+\arctan(3)$

**May 09 (P1)**

A triangle has sides of length $(n^2+n+1)$, $(2n+1)$ and $(n^2-1)$ where $n>1$.

(a) Explain why the side $(n^2+n+1)$ must be the longest side of the triangle.

(b) Show that the largest angle, $\theta$, of the triangle is $120^\circ$.

**May 09 (P2)**

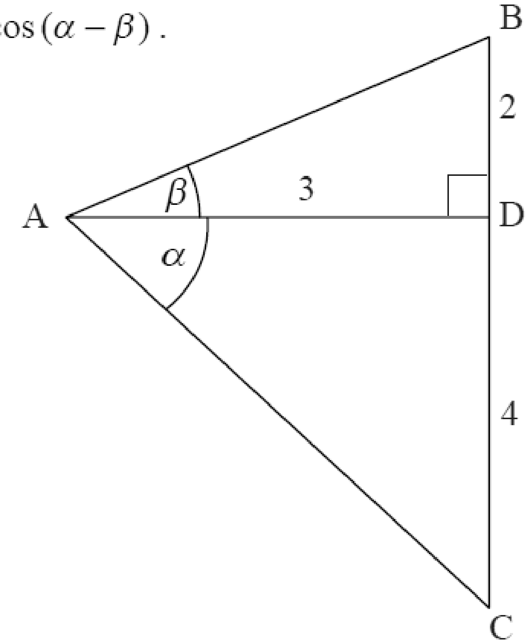
Given that $\boldsymbol{a}=2\sin\theta\,\boldsymbol{i}+(1-\sin\theta)\,\boldsymbol{j}$, find the value of the acute angle $\theta$, so that $\boldsymbol{a}$ is perpendicular to the line $x+y=1$.

**Nov 08 (P2)**

In a triangle ABC, $\hat{A}=35^\circ$, $BC=4$ cm and $AC=6.5$ cm. Find the possible values of $\hat{B}$ and the corresponding values of AB.

**Mayo 08 (P1)**

In the diagram below, AD is perpendicular to BC.
$CD=4$, $BD=2$ and $AD=3$. $C\hat{A}D=\alpha$ and $B\hat{A}D=\beta$.
Find the exact value of $\cos(\alpha-\beta)$.




**Muestra 08 (P1)**

Solve $\sin 2x=\sqrt{2}\cos x$, $0\le x\le\pi$.

**Muestra 08 (P1)**

(a) If $\sin(x-\alpha)=k\sin(x+\alpha)$ express $\tan x$ in terms of $k$ and $\alpha$.

(b) Hence find the values of $x$ between $0^\circ$ and $360^\circ$ when $k=\frac{1}{2}$ and $\alpha=210^\circ$

**Muestra 08 (P1)**

The angle $\theta$ satisfies the equation $2\tan^2\theta-5\sec\theta-10=0$, where $\theta$ is in the second quadrant. Find the value of $\sec\theta$.

**Muestra 08 (P1)**

The obtuse angle $B$ is such that $\tan B = -\frac{5}{12}$. Find the values of

(a) $\sin B$;

(b) $\cos B$;

(c) $\sin 2B$;

(d) $\cos 2B$.

**Muestra 08 (P2)**

The lengths of the sides of a triangle ABC are $x-2$, $x$ and $x+2$. The largest angle is $120^\circ$.

(a) Find the value of $x$.

(b) Show that the area of the triangle is $\frac{15\sqrt{3}}{4}$.

(c) Find $\sin A + \sin B + \sin C$ giving your answer in the form $\frac{p\sqrt{q}}{r}$ where $p, q, r \in \mathbb{Z}$.

**Muestra 08 (P2)**

A farmer owns a triangular field ABC. The side [AC] is 104 m, the side [AB] is 65 m and the angle between these two sides is $60^\circ$.

(a) Calculate the length of the third side of the field.

(b) Find the area of the field in the form $p\sqrt{3}$, where $p$ is an integer.

Let D be a point on [BC] such that [AD] bisects the $60^\circ$ angle. The farmer divides the field into two parts by constructing a straight fence [AD] of length $x$ metres.

(c) (i) Show that the area of the smaller part is given by $\frac{65x}{4}$ and find an expression for the area of the larger part.

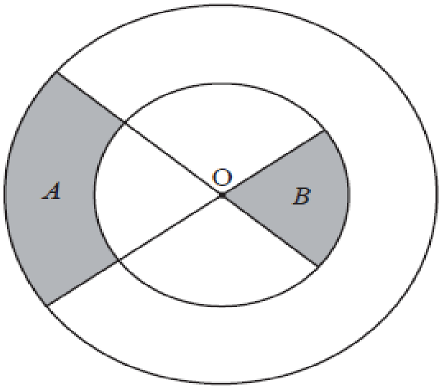
(ii) Hence, find the value of $x$ in the form $q\sqrt{3}$, where $q$ is an integer.

(d) Prove that $\frac{\text{BD}}{\text{DC}} = \frac{5}{8}$.

# Problemas Variados en exámenes BI-NS

**May 09 (P1)**

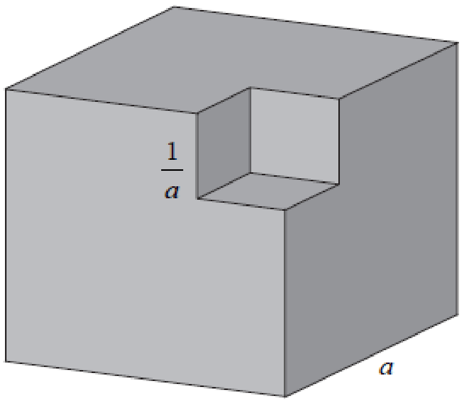
The diagram below shows two straight lines intersecting at O and two circles, each with centre O. The outer circle has radius $R$ and the inner circle has radius $r$.




Consider the shaded regions with areas $A$ and $B$. Given that $A:B = 2:1$, find the **exact** value of the ratio $R:r$.

**May 09 (P1)**

The diagram below shows a solid with volume $V$, obtained from a cube with edge $a > 1$ when a smaller cube with edge $\frac{1}{a}$ is removed.




Let $x = a - \frac{1}{a}$.

(a) Find $V$ in terms of $x$.

(b) Hence or otherwise, show that the only value of $a$ for which $V = 4x$ is $a = \frac{1+\sqrt{5}}{2}$.

**May 09 (P2)**

Six people are to sit at a circular table. Two of the people are not to sit immediately beside each other. Find the number of ways that the six people can be seated.

**Nov 08 (P1)**

Three distinct non-zero vectors are given by $\vec{OA} = \boldsymbol{a}$, $\vec{OB} = \boldsymbol{b}$, and $\vec{OC} = \boldsymbol{c}$.

If $\vec{OA}$ is perpendicular to $\vec{BC}$ and $\vec{OB}$ is perpendicular to $\vec{CA}$, show that $\vec{OC}$ is perpendicular to $\vec{AB}$.

**Mayo 08 (P1)**

Given any two non-zero vectors $\boldsymbol{a}$ and $\boldsymbol{b}$, show that $|\boldsymbol{a}\times\boldsymbol{b}|^2 = |\boldsymbol{a}|^2|\boldsymbol{b}|^2 - (\boldsymbol{a}\bullet\boldsymbol{b})^2$